Lecture 3
Unit root modelling

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1

1

Structure

- Modelling nonstationary time series: unit root time series
- Unit root tests

2

2

Modelling nonstationary time series: unit root time series

3

3

Structure

- Nonstationarity in economic time series
- Special model: random walk
- Why should we be aware of unit roots?
- Different interpretation of random walk
- ARIMA models

4

4

Nonstationarity in economic time series

- For economic time series nonstationarity behaviour is often the most dominant characteristic.
  - Some series grow in a secular way over long periods of time
    - This is evident in time series that measure aggregate economic behaviour (prices, GDP, wages, etc.)
  - Some series appear to wander around as if they have no fixed mean
    - This is evident in many financial time series (interest rates and asset prices).
- Nonstationarity is usually described by the presence of unit roots.

5

5

The simplest form of nonstationarity
Random walk (RW)

$X_t = X_{t-1} + a_t,$

$\mathrm{E}(\mathrm{a}_t) = 0,\ \mathrm{var}(\mathrm{a}_t) = \sigma_a^2,\ \mathrm{cov}(\mathrm{a}_t, a_{t-l}) = 0$

$$X_t = \underbrace{X_{t-1}}_{X_{t-2}+a_{t-1}} + a_t$$

$$X_t = a_t + a_{t-1} + X_{t-2} = a_t + a_{t-1} + a_{t-2} + X_{t-3} = \ldots$$

$$X_t = \underbrace{a_t + a_{t-1} + a_{t-2} + \ldots + a_1}_{\mathrm{t}} + \underbrace{X_0}_{0}$$

$$X_t = a_t + a_{t-1} + a_{t-2} + \ldots + a_1$$

6

6

Random walk II

$X_1 = a_1, \mathrm{var}(X_1) = \mathrm{var}(a_1) = \sigma_a^2$
$X_2 = a_2 + a_1, \mathrm{var}(X_2) = \mathrm{var}(a_2 + a_1) = 2\sigma_a^2$
$\dots$
$\mathrm{var}(X_t) = \mathrm{var}(a_t + a_{t-1} + a_{t-2} + \dots + a_1)$
$= \underbrace{\sigma_a^2 + \sigma_a^2 + \dots + \sigma_a^2}_{t}$
$\mathrm{var}(X_t) = t\sigma_a^2.$

7

7

Random walk III

$X_t = a_t + a_{t-1} + \dots + a_1$
$X_{t-l} = a_{t-l} + a_{t-l-1} + \dots + a_1$

- $\mathrm{cov}(X_t, X_{t-l}) = E\big(X_t - E(X_t)\big)\big(X_{t-l} - E(X_{t-k})\big)$
  $= E(a_t + a_{t-1} + \dots + a_{t-l} + a_{t-l-1} + \dots + a_1)$
  $(a_{t-l} + a_{t-l-1} + \dots + a_1)$
  $=(t-l)\,\sigma_a^2$
- $\rho = \dfrac{\mathrm{cov}(X_t, X_{t-l})}{\sqrt{\mathrm{var}(X_t)\mathrm{var}(X_{t-l})}} = \dfrac{(t-l)\sigma_a^2}{\sqrt{t\sigma_a^2\,(t-l)\sigma_a^2}} = \sqrt{\dfrac{(t-l)^2}{t\,(t-l)}}$

$\rho = \sqrt{1 - \dfrac{l}{t}}$

8

8

Random walk IV

- Random walk does not have a constant variance
  - Variance is linear function of time
  - Variance grows with time
- Covariance between random walk elements depends on time and it is growing with time.
- This is AR(1) model with autoregressive parameter that is equal to 1:
  - ACF decays slowly, starting from the value close to 1.
  - PACF has only one non-zero value: the lag-one partial autocorrelation coefficient.

9

9

Random walk V

- Random walk is transformed into stationary time series by applying once the first difference operator.
- The first difference operator applied once:

$$\Delta X_t = X_t - X_{t-1}$$

- The first difference operator applied twice: the second difference: $\Delta^2 X_t = \Delta\Delta X_t = \Delta X_t - \Delta X_{t-1} = X_t - 2X_{t-1} + X_{t-2}$

$$RW: X_t = X_{t-1} + a_t \Rightarrow$$

$$\underbrace{X_t - X_{t-1}}_{\Delta X_t} = a_t, \Delta X_t = a_t$$

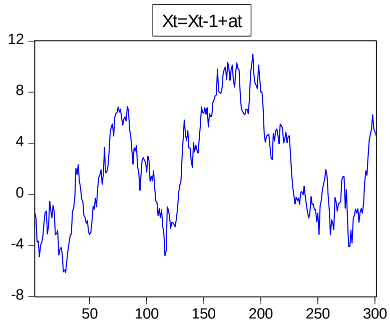
$$E(\Delta X_t) = E(a_t) = 0$$

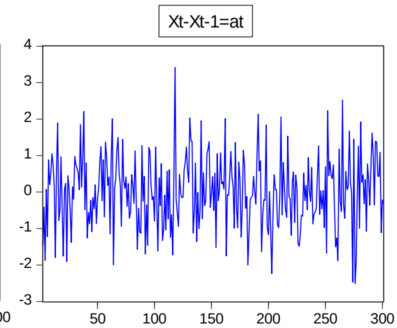
$$\text{var}(\Delta X_t) = \text{var}(a_t) = \sigma_a^2 = \text{const}, t = 1,2,\ldots$$

10

10

Random walk: an example of generated time series

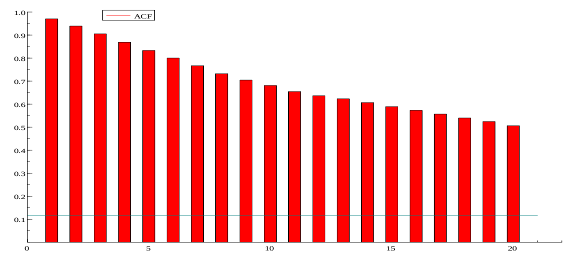


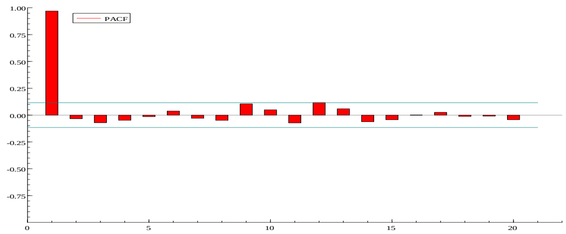


11

11

Random walk: ACF and PACF





12

12

Random walk with drift

$X_t = b + X_{t-1} + a_t, \qquad b > 0$, drift

$$X_t = b + \underbrace{X_{t-1}}_{b+X_{t-2}+a_{t-1}} + a_t$$

$$X_t = 2b + a_t + a_{t-1} + X_{t-2} = \cdots$$

$$X_t = bt + \underbrace{a_t + a_{t-1} + \ldots + a_1}_{t} + \underbrace{X_0}_{0}$$

$$\boldsymbol{X_t = bt + a_t + a_{t-1} + \ldots + a_1}$$

$t = 1, X_1 = b + a_1,$
$t = 2, X_2 = 2b + a_2 + a_1,$ etc.

Deterministic component is augmented by value b each period.

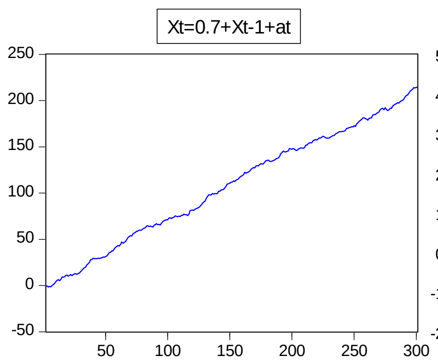
Again the first difference operator is applied to eliminate nonstationarity:

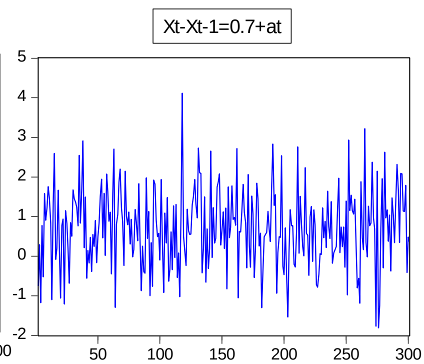
$\Delta X_t = b + a_t$, $E(\Delta X_t) = b$, $var(\Delta X_t) = var(a_t) = \sigma_a^2$.

13

13

Random walk with drift II





14

14

The Random walk model of stock prices

- Today´s (ln) price is just yesterday´s (ln) price plus a random shock.
- The rate of return is white noise. Stock returns are uncorrelated over time.
- This model is a slight oversimplification of the true behaviour of stock prices.
  - Over time the prices of most stocks tend to drift upward, so that the mean rate of return is positive (RW with drift)
- The Efficient Markets Hypothesis:
  - All available information is fully reflected in prices.
  - The price changes due to the arrival of new information.
  - Memory is irrelevant.

$$\ln P_t = \ln P_{t-1} + a_t \Rightarrow \ln P_t - \ln P_{t-1} = \underbrace{\Delta \ln P_t}_{\ln \text{return}} = a_t$$

15

15

Why should we be aware of nonstationarity (unit roots)?

- Statistical point of view
- Economic point of view

16

16

Why should we be aware of nonstationarity (unit roots)? **Statistical point of view**

- The application of standard linear regression model and standard statistical techniques (the OLS method) may induce invalid conclusions if variables employed in regression models are nonstationary.

17

17

Why should we be aware of nonstationarity (unit roots)? **Statistical point of view II**

- The OLS parameter estimators are biased and inconsistent.
- The OLS parameter estimators are not normally distributed.

  The usual t-ratios do not have t-distribution and the sual F-statistics do not have F-distribution.
- Spurious regressions (non-sense correlation) are possible: high coefficient of determination, R^2, is likely to be achieved even with mutually uncorrelated data

18

18

Relevant researches on spurious regression

- **Yule (1926)**
  - Empirical analysis, England and Wales, annual data,1866-1911, Church of England marriages (relative to all marriages) and the standardized mortality (per 1000 persons)
  - R2=**0.91**
- **Granger and Newbold (1974)**
  - Extensive Monte Carlo simulations
- **Hendry (1980)**
  - Empirical analysis: Great Britain, quarterly data, 1964-1975. The UK inflation rate is ˝explained perfectly˝ by rainfall
  - R2=**0.99**
- **Phillips (1986)**
  - Theoretical derivations and explanations

19

19

Monte Carlo simulation

Number of replications 1000, sample size 150, Coefficient of determination, R^2

```
workfile spurious_reg  u 1000
series r2

!nreps=1000
!nobs=150
for !repc=1 to !nreps

smpl @first @first
series y=0
series x=0

smpl @first+1 !nobs+20
'Two unrelated white noise processes'
series ay=nrnd
series ax =nrnd
```

```
'Two unrelated RWs'
series y=y(-1)+ay
series x=x(-1)+ax

smpl @first+20 !nobs+20
equation eq1.ls y c x

'Coeff. of determination R2'
r2(!repc)=@r2

next
smpl @first !nreps
```

20

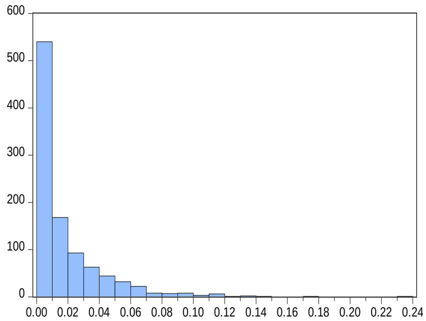
Average R^2

Experiment	Time series	Average R^2
1.	Two unrelated stationary time series X_t=0.6*X_{t-1}+ax_t ,Y_t=0.7*Y_{t-1}+ay_t	0.02
2.	Two correlated stationary time series X_t=0.6*X_{t-1}+ax_t ,Y_t=1+X_t+ay_t	0.60
3.	Two unrelated random walks X_t=X_{t-1}+ax_t ,Y_t=Y_{t-1}+ay_t	0.23
4.	Two unrelated random walks with drift X_t=0.1+X_{t-1}+ax_t ,Y_t=0.2+Y_{t-1}+ay_t	0.49
4a.	Two unrelated random walks with drift X_t=0.5+X_{t-1}+ax_t ,Y_t=0.2+Y_{t-1}+ay_t	0.80

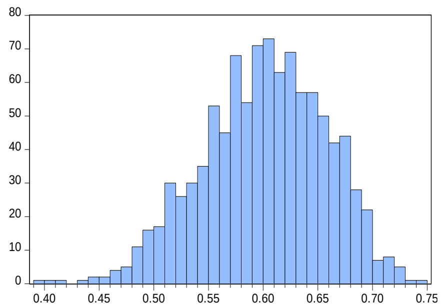
21

Experiments 1 and 2 Distribution of R^2

Exp. 1



Exp. 2

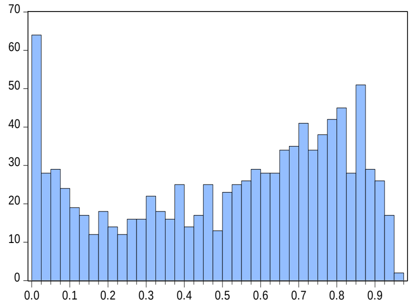


22

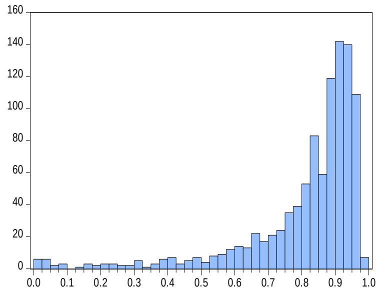
22

Experiments 4 and 4a
Distribution of R^2

Exp. 4



Exp. 4a



23

23

Why should we be aware of nonstationarity (unit roots)? **Economic point of view**

- Relevant issue: Does a shock to a certain time series have a permanent effect or not?
  - **If time series is stationary, then these effects will die out over time** (remember linear process representation, here given for AR(1))

$$X_t = a_t + \phi_1 a_{t-1} + \phi_1^2 a_{t-2} + \phi_1^3 a_{t-3} + \dots$$

  - **If time series is nonstationary, then these shocks will have non-vanishing effect over time**

$$X_t = a_t + a_{t-1} + a_{t-2} + a_{t-3} + \cdots + a_1$$

24

24

Why should we be aware of nonstationarity (unit roots)? **Economic point of view II**

- Example from the business cycle theory:

Do economic recessions have *permanent consequences* for the level of future real GDP or instead they represent *temporary downturns* with the lost output eventually made up during the recovery?

25

25

Other terms used for random walk:

- Integrated time series
- Time series with unit root

26

26

- Integrated time series

  - Random walk is a sum of white noise terms.
  - Operation of summation is equivalent to operation of integration.
  - It is time series **integrated of *order one***, because it is given as ***one*** sum of random elements.
  - ***Order one*** suggests that first differencing needs to be employed *once* to attain stationarity.

27

27

- Integrated time series II

  - Notation: X_t~*I(1)*
  - Stationary time series is integrated of order zero.
    - Notation: a_t~*I(0),*
    - *If* X_t~*I(1) then* ΔX_t~*I(0).*

28

28

- <u>Time series with unit root</u>
  - AR(1) model with autoregressive parameter that is equal to one. The path of time series is determined by the solution of the following characteristic equation:

$$X_t - 1 \cdot X_{t-1} = a_t$$
$$g - 1 = 0 \Rightarrow g = 1.$$

  - The root (solution) is exactly one. This is where the term unit root comes from.
  - The number of unit roots is equal to the order of integration of time series.

29

29

Summary of terms introduced:

If time series has ***d*** unit roots, then it is integrated of order ***d***, implying that it needs to be differenced at least ***d*** times to become stationary time series.

Time series has d unit roots $\Leftrightarrow X_t \sim I(d) \Leftrightarrow \Delta^d X_t \sim I(0)$

30

30

Time series with two unit roots

$X_t = 2X_{t-1} - X_{t-2} + a_t$
$X_t - 2X_{t-1} + X_{t-2} = a_t$

$g^2 - 2g + 1 = 0 \Leftrightarrow (g-1)^2 = 0 \Rightarrow g_1 = g_2 = 1$
$\Rightarrow X_t \sim I(2)$

$$\underbrace{\underbrace{X_t - X_{t-1}}_{\Delta X_t} = \underbrace{X_{t-1} - X_{t-2}}_{\Delta X_{t-1}} + a_t}_{\Delta X_t \sim I(1)} \Rightarrow \underbrace{\underbrace{\underbrace{\Delta X_t = \Delta X_{t-1}}_{\Delta X_t - \Delta X_{t-1} = \Delta^2 X_t}}_{\Delta^2 X_t = a_t}}_{\Delta^2 X_t \sim I(0)} + a_t$$

31

31

Time series with two unit roots: double sum of random shocks

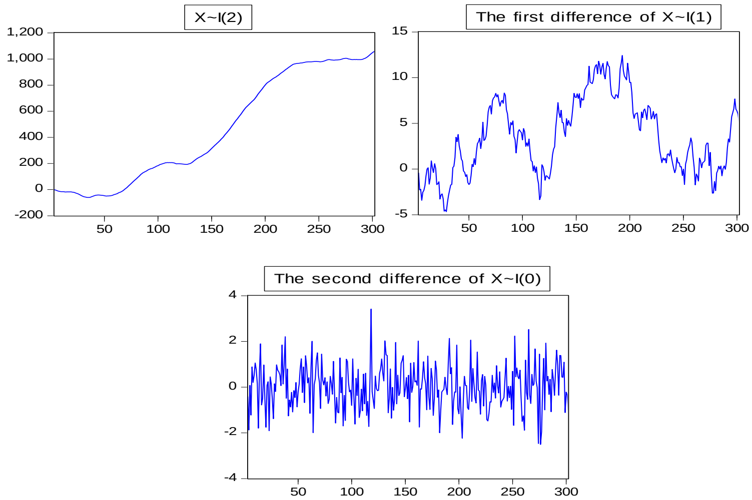
$X_t = 2X_{t-1} - X_{t-2} + a_t$

$$X_t = \underbrace{a_t + 2a_{t-1} + 3a_{t-2} + \ldots + (t-1)a_2 + ta_1}_{\sum_{s=1}^{t} \sum_{j=1}^{s} a_j}$$

32

32

Time series with two unit roots: generated data



33

33

General class of models: Autoregressive integrated moving average models ARIMA(p,d,q) models

$$\Delta^d X_t = \phi_0 + \phi_1 \Delta^d X_{t-1} + \phi_2 \Delta^d X_{t-2} + \ldots + \phi_p \Delta^d X_{t-p}$$
$$+ a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \theta_q a_{t-q}$$

- *p* is the order of autoregressive component
- *d* is the order of integration of time series and
- *q* is the order of moving average component.

34

34

ARIMA(p,d,q) models: Examples

ARIMA(p,1,q):
$$\Delta X_t = \phi_0 + \phi_1 \Delta X_{t-1} + \phi_2 \Delta X_{t-2} + \ldots + \phi_p \Delta X_{t-p} + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \theta_q a_{t-q}$$

ARIMA(p,2,q):
$$\Delta^2 X_t = \phi_0 + \phi_1 \Delta^2 X_{t-1} + \phi_2 \Delta^2 X_{t-2} + \ldots + \phi_p \Delta^2 X_{t-p} + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \theta_q a_{t-q}$$

AR(p)	MA(q)	ARMA(p,q)	White noise	Random walk
ARIMA(p,0,0)	ARIMA(0,0,q)	ARIMA(p,0,q)	ARIMA(0,0,0)	ARIMA(0,1,0)

35

35

Unit root tests

36

36

Introduction

- The application of the unit root tests has the following two goals.
  - To determine whether time series has a unit root or not.
  - To determine exact number of unit roots.
- There are many unit root tests
  - Dickey-Fuller (DF) test, 1976.
  - Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test, 1992.
  - Elliott-Rothenberg-Stock (ERS or DF-GLS) test, 1996.
- Choi (2015), *Almost all about unit roots,* Cambridge University Press.

37

37

Dickey-Fuller (DF) test

- Basic idea
- The presence of different deterministic components
- How to calculate test-statistic?
- The decision rule
- Monte-Carlo simulation
- How to determine the number of unit roots?
- Augmented DF test
- Drawbacks (KPSS test)
- The Box-Jenkins approach once again

38

38

Basic idea

- Baseline model:
  $X_t = \phi_1 X_{t-1} + a_t$
- Hypotheses:
  H_0: X_t has one unit root, $\phi_1 = 1$, $X_t \sim I(1)$
  H_1: X_t is stationary time series, $\phi_1 < 1$, $X_t \sim I(0)$

- Alternative specification of baseline model:
  $\Delta X_t = (\phi_1 - 1) X_{t-1} + a_t$, $\phi_1 - 1 = \psi_1$
  such that hypotheses are:
  H_0: X_t has one unit root, $\psi_1 = 0$, $X_t \sim I(1)$
  H_1: X_t is stationary time series, $\psi_1 < 0$, $X_t \sim I(0)$

39

39

DF test for different deterministic components

DF test	τ	τ_μ	τ_t
Deterministic components	None	Constant	Constant+ Linear trend
$E(X_t)$	0	μ	$\mu + bt$, $t = 1, 2, \ldots$

40

40

DF test for different deterministic components II

- There are three types of DF test: τ, τ_μ, τ_t.

- Null (H_0) and alternative (H_1) hypotheses:

1. H_0: $X_t = X_{t-1} + a_t$, Series is random walk
   H_1: $X_t = \phi_1 X_{t-1} + a_t$, ϕ_1<1, Series is stationary with zero mean

2. H_0: $X_t = X_{t-1} + a_t$, Series is random walk
   H_1: $X_t = \phi_1 X_{t-1} + constant + a_t$, ϕ_1<1,
   Series is stationary with non-zero mean

3. H_0: $X_t = b + X_{t-1} + a$, Series is random walk with drift (b)
   H_1: $X_t = \phi_1 X_{t-1} + constant + trend + a_t$, ϕ_1<1,
   Series is stationary around linear trend

41

41

What is the relevant model to start with depending on deterministic components?

DF test	Model estimated by the OLS method based on sample of size T
τ	$\Delta X_t = \psi_1 X_{t-1} + a_t$
τ_μ	$\Delta X_t = \psi_1 X_{t-1} + \beta_0 + a_t$
τ_t	$\Delta X_t = \psi_1 X_{t-1} + \beta_0 + \beta t + a_t$

42

42

How to calculate DF test-statistic?

- We apply the OLS method:

$$\Delta\hat{X}_t = \hat{\psi}_1 X_{t-1} + \hat{\beta}_0 + \hat{\beta} t$$
$$s(\hat{\psi}_1)$$

- **DF test-statistic: ratio of estimator of parameter** $\hat{\psi}_1$ **and corresponding standard error**

$$DF = \tau_t = \frac{\hat{\psi}_1}{s(\hat{\psi}_1)}$$

43

43

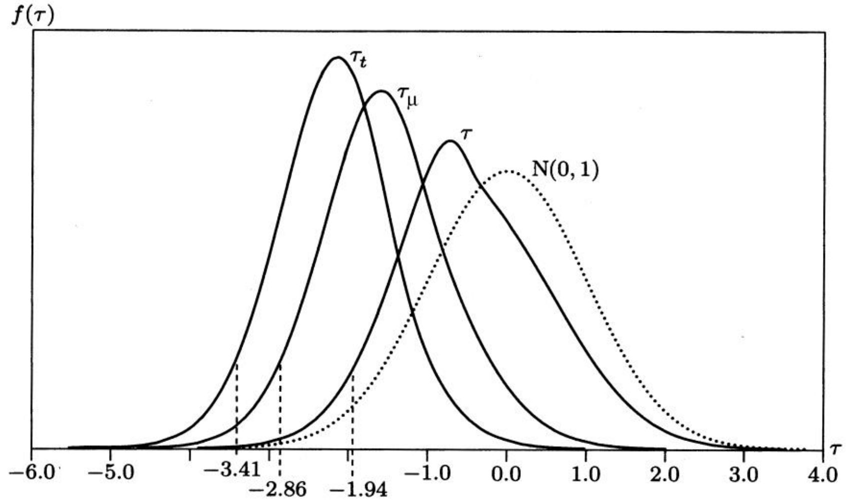
How to calculate DF test-statistic?

- DF has the form of standard *t*-ratio.
- DF **does not have *t*-distribution** when the null is true.
- DF has non-standard limiting distribution.
- Critical values can be derived by simulation methods as first in Fuller (1976).
- It is possible to obtain critical value for every sample size, MacKinnon (1991).

44

44

Density functions of three DF test-statistics for large sample



45

Critical values at the 5% level for given sample size *T* (MacKinnon, 1991)

$$\tau = -1.9393 - \frac{0.398}{T}$$

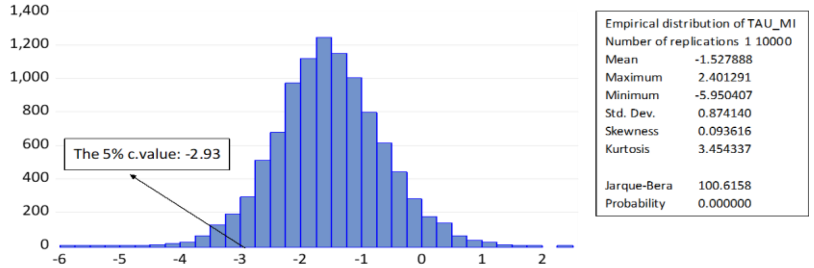
$$\tau_{\mu} = -2.8621 - \frac{2.738}{T} - \frac{8.36}{T^2}$$

$$\tau_{t} = -3.4126 - \frac{4.039}{T} - \frac{17.83}{T^2}$$

46

46

Decision rule (picture is for T=50) $DF = \frac{\hat{\psi}_1}{s(\hat{\psi}_1)}$



- Null hypothesis that unit root exists is not accepted if calculated DF statistic is "sufficiently small" (if it is smaller than the critical value).

- Null hypothesis that unit root exists is accepted if calculated DF statistic is "sufficiently big" (if it is larger than the critical value).

47

47

Determining the number of unit-roots I

- If H_0 is not accepted, then series does not have unit roots. **It is stationary**. End of testing.

- If H_0 is accepted, then series has at least one unit root. **It is integrated of order 1, X_t~I(1).**
  - The testing procedure goes one.
  - It is necessary to find out if the number of unit roots is exactly one or eventually two.

48

48

Determining the number of unit-roots II

- We continue with the testing procedure:

  H_0: $X_t \sim I(2)$ versus H_1: $X_t \sim I(1)$

  H_0: $\Delta X_t \sim I(1)$ versus H_1: $\Delta X_t \sim I(0)$.

- Time series ΔX_t is considered.

  $$\Delta X_t = \phi_1 \Delta X_{t-1} + \beta_0 + \beta t + a_t \ /- \Delta X_{t-1}$$

  $$\Delta \Delta X_t = \psi_1 \Delta X_{t-1} + \beta_0 + \beta t + a_t, \ \phi_1 - 1 = \psi_1$$

  $$\Delta^2 X_t = \psi_1 \Delta X_{t-1} + \beta_0 + \beta t + a_t.$$

- **We regress $\Delta^2 X_t$ on ΔX_{t-1}, constant and trend and check if t-ratio for estimator of ΔX_{t-1} is smaller or greater than the appropriate critical value of DF test.**

49

49

Determining the number of unit-roots III

- If H_0 is rejected, then series has exactly one unit root, $X_t \sim I(1)$. End of testing.

- If H_0 is not rejected, then series has at least two unit roots. **It is integrated of at least order 2, $X_t \sim I(2)$.**
  - The testing procedure goes one.
  - We should determine whether the number of unit roots is exactly two or maybe three. *Time series $\Delta^2 X_t$ is considered…*

50

50

Augmented DF test: introduction

- If order of AR model is higher than one, then autocorrelation in residuals is present when AR(1) is used.
- Key consequence of autocorrelation in residuals in CLRM: standard errors are invalid.
- DF test in the presence of autocorrelation in residuals
  - UNRELIABLE
  - Denominator of DF statistic is wrong

51

51

Augmented DF test, ADF(***K***)

- $\Delta X_t = \beta_0 + \beta t + \psi_1 X_{t-1} + \delta_1 \Delta X_{t-1} + \delta_2 \Delta X_{t-2} + \ldots + \delta_{\boldsymbol{K}} \Delta X_{t-\boldsymbol{K}} + a_t$
- ADF test–statistic: the OLS estimator of ψ_1 divided by corresponding standard error.
- ADF and DF tests have the same limiting distributions: the same critical values may be applied.
- Parameter ***K*** can be determined in different ways:
  - ‘Specific to general’ approach (enlarge one by one)
  - ‘General to specific’ approach (enlarge with 4 or 12 lags and reduce number of lags if necessary)
  - Using information criteria (AIC, SC, HQC)

52

52

Main criticism of DF test

- The power of DF test is low if time series is stationary but with a root close to the non-stationary boundary. Test is poor at deciding if

$$\phi_1=1 \text{ or } \phi_1=0.90,$$

especially with small sample sizes.

- If the true data generating process is

$$X_t = 0.90X_{t-1} + a_t$$

then the null hypothesis of a unit root should be rejected, but with DF test that is not always the case.

- One way to get around this is to use a unit root test with the null hypothesis of stationarity. One such test is KPSS test (Kwiatkowski, Phillips, Schmidt and Shin, 1992) 53

53

Main criticism of DF test II

- The power of DF test is unreliable if structural break exists.

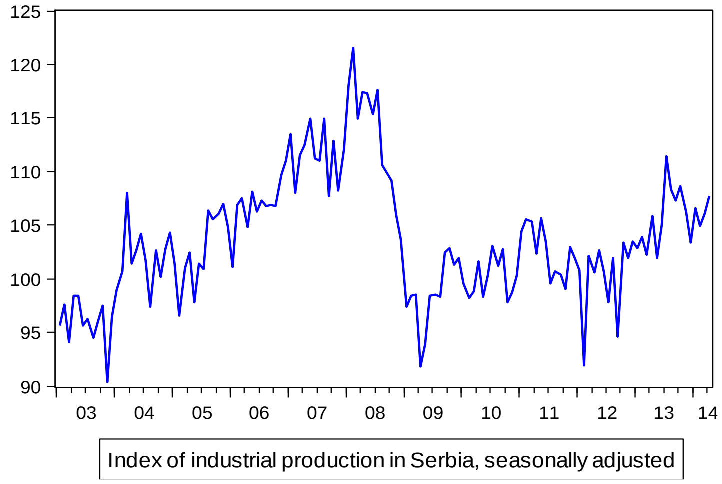
- If time series is trend stationary with permanent structural break, then DF test is biased towards wrongly accepting the null of a unit root.

  - Different set of unit root tests is designed to account for the problem.

54

54

Example
Serbia: 2003:1 – 2014:4



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To conclude

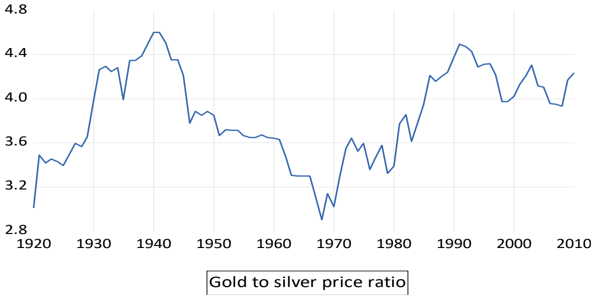
- The Box-Jenkins methodology is focused on finding out **ARIMA** specification that fits the data well.
- In the first stage of identification, we also need to identify ***d***, i.e. the order of integration (the number of unit roots).
- Therefore, unit root tests are part of the standard procedure used to identify appropriate ARIMA model.

56

56

Example 1: Gold to silver price ratio (per ounce)

- Period: 1920 – 2010
- Annual observations (91), log values, ur1.wf1



57

57

Example 1: DF testing

$$\hat{X}_t = 0.349 + \underset{(0.038)}{0.913} X_{t-1}$$

$$\Delta\hat{X}_t = 0.008 + \underset{(0.101)}{0.034} \Delta X_{t-1}$$

I step

H_0: $X_t \sim I(1)$, $\phi_1 = 1$

H_1: $X_t \sim I(0)$, $\phi_1 < 1$

$$\left.\begin{array}{l} DF = \dfrac{0.913 - 1}{0.038} = -2.29 \\ \tau_\mu^k = -2.89 \ (\alpha = 0.05) \end{array}\right\} \Rightarrow \left\{\begin{array}{l} -2.29 > -2.89 \\ H_0 \text{ is not rejected.} \\ \text{Series has at least one unit root.} \end{array}\right.$$

II step

H_0: $X_t \sim I(2) \Leftrightarrow \Delta X \sim I(1)$

H_1: $X_t \sim I(1) \Leftrightarrow \Delta X \sim I(0)$

$$\left.\begin{array}{l} DF = \dfrac{0.034 - 1}{0.101} = -9.56 \\ \tau_\mu^k = -2.89 \ (\alpha = 0.05) \end{array}\right\} \Rightarrow \left\{\begin{array}{l} -9.56 < -2.89 \\ H_0 \text{ is rejected.} \\ \text{Series has one unit root.} \end{array}\right.$$

58

58

Example 1: DF testing within an alternative model

We considered the following results:

$$\hat{X}_t = 0.349 + 0.913X_{t-1}$$
$$(0.038)$$
$$\Delta\hat{X}_t = 0.008 + 0.034\Delta X_{t-1}$$
$$(0.101)$$

They can be given in the modified form (X_{t-1} is subtracted from both sides):

$$\widehat{\Delta X}_t = 0.349 - 0.087\, X_{t-1}$$
$$(0.038)$$
$$\widehat{\Delta^2 X}_t = 0.008 - 0.966\, \Delta X_{t-1}$$
$$(0.101)$$

59

59

Example 1: DF testing within an alternative model

I step

$$\widehat{\Delta X}_t = 0.349 - 0.087\, X_{t-1}$$
$$(0.038)$$
$$\widehat{\Delta^2 X}_t = 0.008 - 0.966\, \Delta X_{t-1}$$
$$(0.101)$$

I step

H_0: $X_t \sim I(1)$, $\psi_1 = 0$

H_1: $X_t \sim I(0)$, $\psi_1 < 0$

$$\left.\begin{array}{l} DF = -\dfrac{0.087}{0.038} = -2.29 \\ \tau_{\mu}^{k} = -2.89\ (\alpha = 0.05) \end{array}\right\} \Rightarrow \begin{cases} -2.29 > -2.89 \\ H_0 \text{ is not rejected.} \\ \text{Series has at least one unit root.} \end{cases}$$

60

60

Example 1: DF testing within an alternative model
II step

$$\widehat{\Delta X_t} = 0.349 - 0.087\, X_{t-1}$$
$$(0.038)$$
$$\widehat{\Delta^2 X_t} = 0.008 - 0.966\, \Delta X_{t-1}$$
$$(0.101)$$

II step

H_0: $X_t \sim I(2) \Leftrightarrow \Delta X \sim I(1)$
H_1: $X_t \sim I(1) \Leftrightarrow \Delta X \sim I(0)$

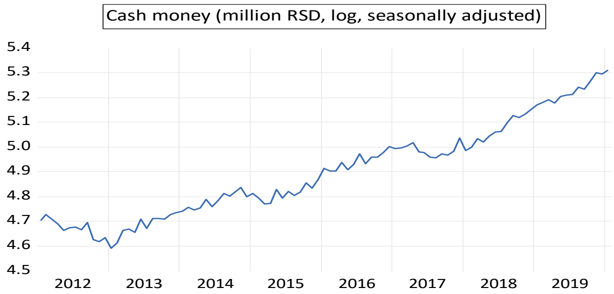
$$\left.\begin{matrix} DF = -\dfrac{0.966}{0.101} = -9.56 \\ \tau_{\mu}^{k} = -2.89\ (\alpha = 0.05) \end{matrix}\right\} \Rightarrow \begin{cases} -9.56 < -2.89 \\ H_0 \text{ is rejected.} \\ \text{Series has one unit root.} \end{cases}$$

61

61

Example 2: Cash money in Serbia

- Period: 2012m1 – 2020m1
- Monthly observations (97), log values, seasonally adjusted, ur2.wf1



62

62

Example 2: ADF testing

I step: Checking for a unit root	II step: Checking for the second unit root
H$_0$: $X_t \sim I(1)$ H$_1$: $X_t \sim I(0)$ ***ADF(3)=-2.64*** τ_t^k=-3.46 -2.64>-3.46 H$_0$ is not rejected. ***Series has at least one unit root.***	H$_0$: $\Delta X_t \sim I(1)$ H$_1$: $\Delta X_t \sim I(0)$ ***ADF(2)=-4.84*** -4.84<-3.46 H$_0$ is rejected. ***First difference is stationary. Cash money has one unit root.***

63