INTERMEDIATE ECONOMETRICS: PART II

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Short description

- This part of the course is designed to introduce ***econometric time-series*** tools used in finance and to gain understanding of the characteristics of financial data.
- The course covers the application of econometric methods to analyze real economic data by using EVIEWS 12/13 software.

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Literature for the course

- Any textbook of Econometrics and Time Series Analysis that covers the subject
- Tsay, R. S. *Analysis of Financial Time Series*, 2010, Wiley, (3rd edition), Ch. 2 and Ch. 3.
- Brooks, C. *Introductory Econometrics for Finance*, 2019, Cambridge University Press (4th edition), Ch. 6 and Ch. 9.
- Mills, T.C. *Applied Time Series Analysis*, 2019, Academic Press.

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Key time series literature

- Box, G.E.P., G.M. Jenkins, G.C. Reinsel and G. M. Ljung, *Time Series Analysis: Forecasting and Control*, 2015, Wiley, (5th edition)
- Brockwell, P. and R. Davis, *Time Series: Theory and Methods,* 1991, Springer-Verlag
- Hamilton, J., Time Series Analysis, 1994, Princeton University Press
- Lutkepohl, H., *A New Introduction to Multiple Time Series Analysis,* 2005, Springer-Verlag

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Types of data

- **Time series data**
  - Anually, quarterly, monthly, daily, as transaction occurred.
- **Cross section data**
  - Data of more variables collected at one point in time.
- **Panel data**
  - Time series data collected for different individuals.

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Our main interest

- Time series data
  - *Collection of data is well organized and these data are easily available*
  - *Methodological framework of econometric time series analysis is well developed.*
  - *Analysis is relevant, especially in forecasting macroeconomic and financial data.*

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WHAT IS TIME SERIES ANALYSIS?

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Key property of time series: autocorrelation

- Time series is the sequence of the data ordered/determined by calendar time.
- Calendar time: year, month, day, hour, a minute,…
  - *Example: Observation on prices in December 2022 precedes observation in January 2023 and later.*
  - *This leads to a consecutive analysis of time series data while leaving their sequence intact.*

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Key property of time series: autocorrelation (II)

- Standard notation: Y_t, $t=1,2,\ldots,T$
  - *t – time index runs from 1 to T and T is the sample size*
  - *It is short-hand for observations:* $Y_1, Y_2, \ldots, Y_T$.
- It is likely that part of the value of Y_{t-1} is reflected in the value Y_t: it makes sense to analyze Y_{t-1} prior to analyzing Y_t
  - *Observations are likely to be correlated.*
  - *The concept is called* ***autocorrelation.***
- Autocorrelation can be exploited to obtain a first impression of possible useful model to describe and forecast the time series.

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Main purpose of time series analysis:

To discover underlying autocorrelation pattern of the data

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Basic difference between standard econometric approach and time series approach

- Standard econometric approach:
  $Y=f(X_1, X_2, \ldots)$, where $X_1, X_2, \ldots$, are variables suggested by economic theory.
- Time series approach:
  $Y_t=f(Y_{t-1}, Y_{t-2}, \ldots)$,
  - *Explanatory variables suggested by economic theory are ignored*
  - *The past behaviour of Y_t is sufficient for modeling*

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Main features of economic time series

- Trend
- Seasonality
- Aberrant observations/structural breaks
- Conditional heteroskedasticity/unstable conditional variability.

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Trend

- Long-term component
- Time series is systematically growing or falling over time.
- Trending can be either stochastic or deterministic.
- Stochastic trend: in time t-1 we cannot forecast value in t.
- Deterministic trend: function of the form a+bt (a,b=const) determines the long-run movement of time series.

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Examples of stochastic trend: Data from the Serbian economy (2014:1 – 2022:10)

Consumer price index (2015=100)

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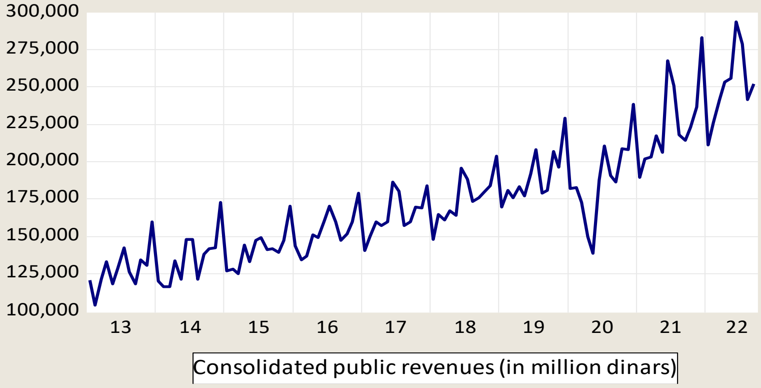
Examples of deterministic trend: Data from the Serbian economy (2013:1 – 2022:10)



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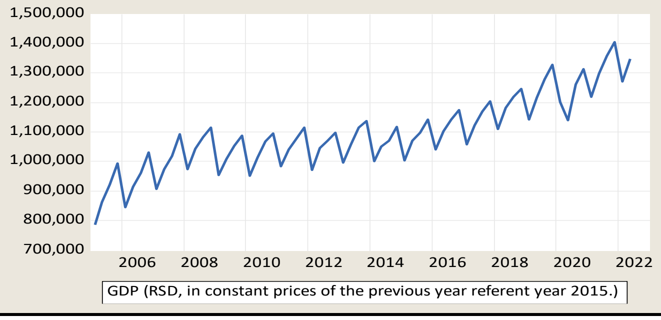
Seasonality

- Quarterly and monthly data
- Similar behaviour in same quarters/month over years
- Deterministic or stochastic
- There is higher correlation between same quarters over different years than between quarters within the given year
- Example: quarterly GDP in Serbia (2005:1 – 2022:2)



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Aberrant observations

- Exogenous events may cause the change in the behavior of time series

  (NATO intervention, transition process, the Great recession, oil price shocks, pandemic, etc.)

- Structural breaks (outliers) – observations that are inconsistent with the rest of the sample
  - *One-time change (additive outlier)*
  - *Persistent change (innovational outlier)*
    - Intercept of trend function (level shift)
    - Slope of trend function (slope shift)
    - Slope and intercept

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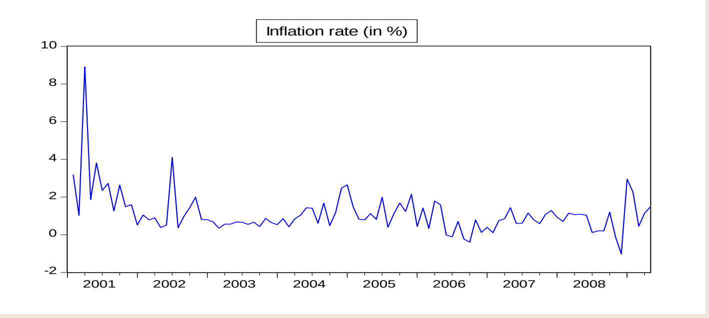
Additive outliers in the Serbian inflation rate during transition process (2001-2009)

- Introduction of extensive tax reform (2001:4)
- Change of the price of electricity (2002:7)
- Introduction of VAT (2005:1)
- Change of several administratively controlled prices (2009:1)



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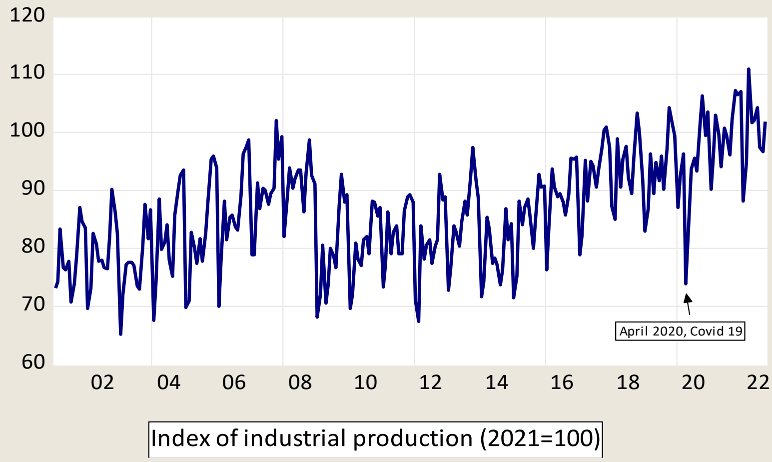
Change in the intercept of trend function in the Serbian index of industrial production (2001:1 – 2022:9) and additive outlier



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Conditional heteroskedasticity

- Key feature of financial time series
- News arrives on a market: we react by selling or buying many stocks. The day after the news was digested: we wish to return to the behaviour before the arrival of the news.
- Pattern: increase or decrease in the returns on one day followed by an opposite change on the next day.
- Large absolute returns tend to appear in clusters.
- Turbulent (high variability) period is followed by quite (low variability) period; these subperiods are recurrent but not in a periodic way.
- Autocorrelation is present, but in data variability
- Unstable conditional variability: conditional heteroskedasticity
- **Conditional variability (or standard deviation): volatility**

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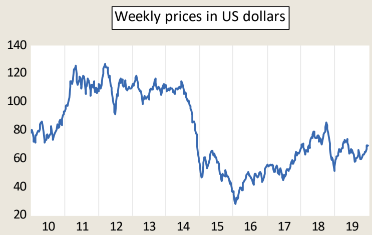
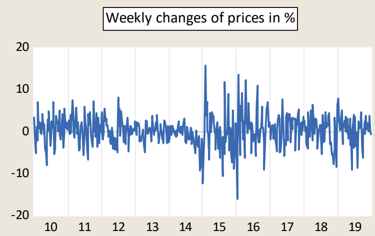
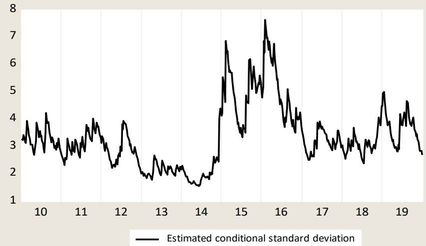
Conditional heteroskedasticity: Weekly changes of oil prices

Europe Brent Spot Price (Dollars per Barrel)

Period: January/I week, 2010 - January/I week, 2020







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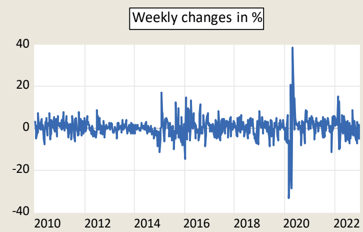
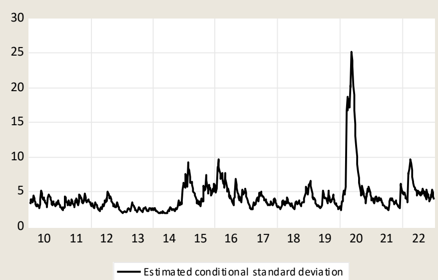
Conditional heteroskedasticity: Weekly changes of oil prices

Europe Brent Spot Price (Dollars per Barrel)

Period: January/I week, 2010 - January/I week, 2023





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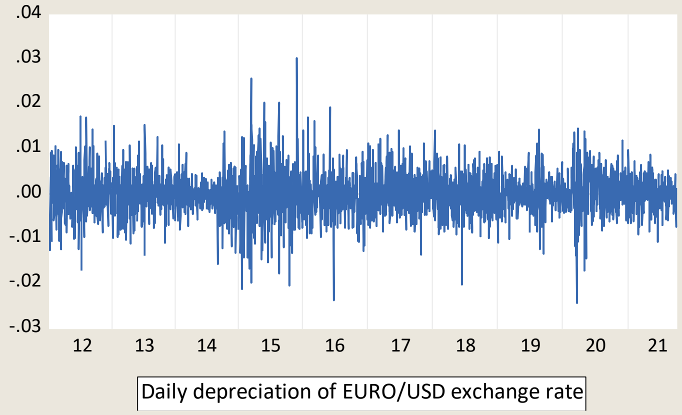
Conditional heteroskedasticity: Daily depreciation of the nominal EURO/USD exchange rate

Period: January 2, 2012 – October 1, 2021



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Is analysis of financial time series different from other time series analysis?

- Not exactly
- But, there is more uncertainty to be captured due to unstable variability
- Time series of interest: asset return
  - *Discrete return (the simple return)*
  - *Continuous return (the log return)*

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The simple asset return

- P_t – the price of an asset at time index t
- P_{t-1} – the price of an asset at time index t-1
- Assumption: this asset pays no dividends
- R_t – *the simple asset (net) return* between t-1 and t:

  $$R_t = (P_t - P_{t-1}) / P_{t-1} = P_t / P_{t-1} - 1$$

  This is also known as *one-period simple return*
- The simple asset return from t-k to t(*k-period return*):

  $$R_t = (P_t - P_{t-k}) / P_{t-k}$$

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Log asset return

- P_{t-1} – the price of an asset at time index t-1
- P_t – the price of an asset at time index t
- Assumption: this asset pays no dividends
- r_t – *the log asset return* between t-1 and t (*one- period log return*): $r_t = \log P_t - \log P_{t-1}$
- If $P_t / P_{t-1} - 1$ is close to zero, then:

  $$r_t = \log P_t - \log P_{t-1} = \log (P_t / P_{t-1}) \approx P_t / P_{t-1} - 1 = R_t$$

  Log asset return ≈ Simple asset return

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Log(1+Δ)≈ Δ for Δ close to zero

- Taylor series expansion of function f(x) around x_0 of order k (f(x) is continuous in the neighborhood of x_0 and its first k derivatives exist)

$$f(x) = f(x_0) + \frac{(x - x_0)}{1!} f'(x_0) + \frac{(x - x_0)^2}{2!} f''(x_0) + \ldots + \frac{(x - x_0)^k}{k!} f^{('k)}(x_0).$$

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Log(1+Δ)≈ Δ (II)

- Taylor series expansion of order 1:

$$f(x) \approx f(x_0) + \frac{(x - x_0)}{1!} f'(x_0)$$

$$f(x) = \log(1 + \Delta)$$

$$\log(1 + \Delta) \approx \log(1) + \Delta \frac{1}{x}\Big|_{x=1} = \Delta.$$

$$\Delta = \frac{P_t - P_{t-1}}{P_{t-1}},$$

$$\log\left(1 + \frac{P_t - P_{t-1}}{P_{t-1}}\right) = \underbrace{\log \frac{P_t}{P_{t-1}}}_{r_t} \approx \underbrace{\frac{P_t - P_{t-1}}{P_{t-1}}}_{R_t}.$$

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Outline of the course

- Basic concepts in univariate time series analysis
- Linear time series models (ARMA models)
- Nonstationarity and unit roots (ARIMA models)
- Modeling conditional variability ((G)ARCH models)

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Outline of the second part of Lecture 1: Basic concepts

- Time series
- Stationarity
- White noise
- Basic tools: autocovariance and autocorrelation functions
- Testing for autocorrelation in time series
- Examples
- Linear process

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Lecture 1
Basic concepts

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Structure

- Elementary notation
- Time series
- Stationarity
- Autocovariance function
- Autocorrelation function
- Autocorrelation tests
- Examples
- Linear process

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Elementary notation

- Time series in time t: r_t
- Lagged-one period: $t-1$
- The lag shift operator: $L\, r_t = r_{t-1}$
  (the backward shift operator: $B\, r_t = r_{t-1}$)

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Elementary notation II

- First order difference operator (ordinary difference):

$$\Delta r_t = r_t - r_{t-1} = (1 - L) r_t$$

- Difference of order k operator (seasonal difference):

$$\Delta_k r_t = r_t - r_{t-k} = \left(1 - L^k\right) r_t$$

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More on the lag shift operator L: some properties

- $L^k r_t = r_{t-k}$, $k=1,2,\ldots$
- $L^{-k} r_t = r_{t+k}$, $k=1,2,\ldots$
- $L\mu = \mu$, $\mu = const.$
- $L^0 r_t = r_t$.
- $L^i (L^j r_t) = L^j (L^i r_t) \Longrightarrow r_{t-j-i} = r_{t-i-j}$.

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Time series: working definition

- A collection of random variables generated sequentially in time.
- More specific, random variables are considered at different time points.
  - *Time points are equally spaced.*
- Notation: $\{r_t\}, t = 1,2, \ldots$
- In empirical work: time series is a record of values of certain quanitity of interest at different time points.

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Stationarity

- Descriptive explanation:
  - Time series has predictable behaviour over time
  - Statistical properties of time series do not “change much” over time
- Two concepts of stationary time series:
  - *Strictly* (stationarity in narrow sence)
  - *Weakly* (stationarity in wide sence)

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Strictly stationary time series

A time series r_t is said to be strictly stationary if

- the joint distributions of random sequences $(r_{t_1}, r_{t_2}, \dots, r_{t_k})$ and $(r_{t_1+t}, r_{t_2+t}, \dots, r_{t_k+t})$ are the same for all *t.*
  - *k* is an arbitrary positive integer and $(t_1,\dots,t_k)$ is ordered collection of *k* positive integers.
  - **Under strict stationarity the joint distribution of $(r_{t_1}, r_{t_2}, \dots, r_{t_k})$ is invariant under time shift.**

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Weakly stationary time series

A time series r_t is said to be weakly stationary if:

1. $E(r_t) = \mu = const,\ t = 1,2,\ldots$

2. $var(r_t) = E(r_t - \mu)^2 = const,\ t = 1,2,\ldots$

3. $\mathrm{cov}(r_t, r_{t-l}) = E(r_t - \mu)(r_{t-l} - \mu) = \gamma(l) = \gamma_l,$
   $t = 1,2,\ldots,\ l = 1,2,\ldots$

- Instead of 2. and 3. just the following:

$$\mathrm{cov}(r_t, r_{t-l}) = E(r_t - \mu)(r_{t-l} - \mu) = \gamma(l) = \gamma_l,$$
$$t = 1,2,\ldots,\ l = 0,1,2,\ldots$$

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Stationarity: covariance

X, Y - random variables

$$\mathrm{cov}(X,Y) = E(X - E(X))(Y - E(Y))$$

$$X = r_t,\quad E(r_t) = \mu$$
$$Y = r_{t-l},\ E(r_{t-l}) = \mu$$

$$\mathrm{cov}(r_t, r_{t-l}) = E(r_t - \mu)(r_{t-l} - \mu)$$

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Weakly stationary time series II

- Expected value and variance are **invariant** under time shift.
- Covariance depends **only on distance/lag** between two elements in time series.
- For a given lag *l,* the covariance is the same:

$$\mathrm{cov}(r_t, r_{t-l}) = const, \text{ for given } l,\ t = 1,2,\ldots$$

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Weakly stationary time series: covariance is only function of time distance

$$r_1, r_2, r_3, \ldots, r_{51}, r_{52}, r_{53}$$

$$\underbrace{r_1, r_2}_{\gamma_1}, r_3, \ldots, \underbrace{r_{51}, r_{52}}_{\gamma_1}, r_{53}$$

$$r_1, \underbrace{r_2, r_3}_{\gamma_1}, \ldots, r_{51}, \underbrace{r_{52}, r_{53}}_{\gamma_1}$$

$$\underbrace{r_1, \ldots, r_3}_{\gamma_2}, \ldots, \underbrace{r_{51}, \ldots, r_{53}}_{\gamma_2}$$

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Relationship between weakly and strictly stationarity

- Strictly stationary time series is also weakly stationary **if its first two moments are finite.**
- **Weakly stationary time series is not strictly stationary in general.**
  - If weakly stationary time series does not have stable moments of order higher than 2, then it is not strictly stationary.
- Both concepts of stationarity are identical if elements of time series are normally distributed.

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Three types of stationarity depending on the expected value

- Expected value
  - Zero
  - Non-zero
  - Linear trend function (trend-stationarity)

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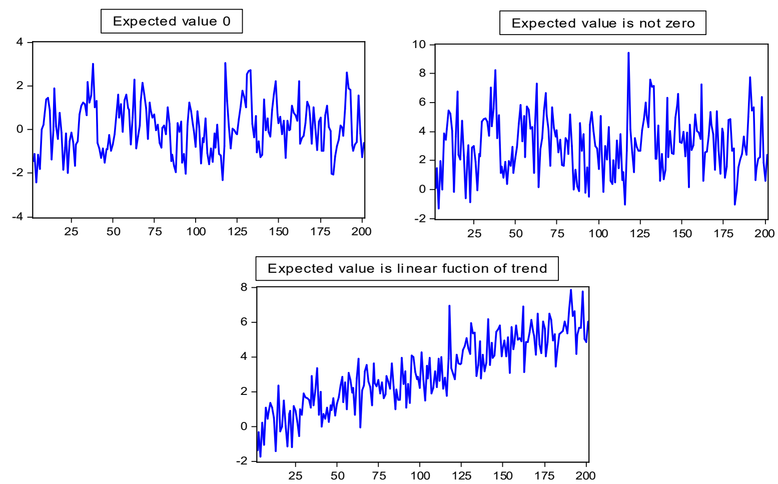
Three types of stationarity depending on the expected value: generated data



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Elementary stationary time series: white noise

1. $E(a_t) = 0, \qquad t = 1,2,\ldots$

2. $\mathrm{var}(a_t) = E(a_t^2) = \sigma_a^2 = const, \ t = 1,2,\ldots$

3. $\mathrm{cov}(a_t, a_{t-l}) = E(a_t a_{t-l}) = 0, \ t = 1,2,\ldots, \ l = 1,2,\ldots$

- Sequence of uncorrelated random variables with zero expected value and finite variance.

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Independent white noise

1. $E(a_t) = 0, \quad t = 1,2,...$

2. $\mathrm{var}(a_t) = E(a_t^2) = \sigma_a^2 = const, \; t = 1,2,...$

3. a_t is sequence of independent random variables (stronger condition)

- Sequence of independent random variables with zero expected value and finite variance.

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Gaussian white noise

1. $E(a_t) = 0, \quad t = 1,2,...$

2. $\mathrm{var}(a_t) = E\left(a_t^2\right) = \sigma_a^2 = const, \; t = 1,2,...$

3. a_t is sequence of independent random variables

4. $a_t : \mathrm{N}(0,\sigma_a^2), \; t = 1,2,...$

- Sequence of independent and normally distributed random variables with zero expected value and finite variance.

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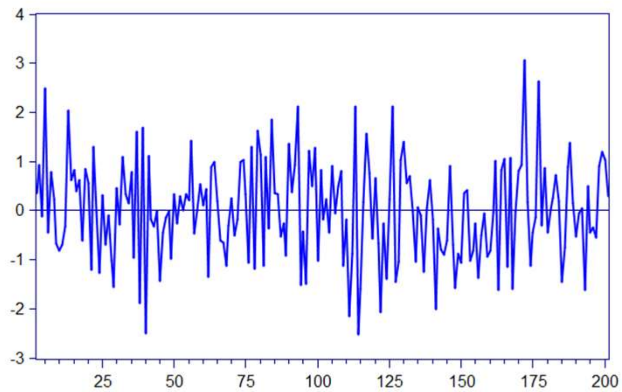
Gaussian white noise: generated data



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Key tools for examining autocorrelation

- The autocovariance function
- The autocorrelation function

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The lag-*l* autocovariance

- The lag-*l* autocovariance:

$$\gamma_l = \text{cov}(r_t, r_{t-l}) = E(r_t - \mu)(r_{t-l} - \mu), l = 0,1,2,\ldots$$

- Two key properties:

1. $\gamma_0 = E(r_t - \mu)^2 = \text{var}(r_t)$
2. $\gamma_l = \gamma_{-l}$

- $\gamma_l = \text{cov}(r_t, r_{t-l}) = \text{cov}(r_{t-l}, r_t) = \text{cov}(r_t, r_{t+l}) = \gamma_{-l}$
- The autocovariance function: $\gamma_1, \gamma_2, \gamma_3, \ldots$

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Autocorrelation function (ordinary)

- The lag-*l* autocorrelation coefficient measures correlation between r_t and r_{t-l}:

$$\rho_l = \frac{\text{cov}(r_t, r_{t-l})}{\sqrt{\text{var}(r_t)\,\text{var}(r_{t-l})}}, l = 0,1,2,\ldots$$

$$\rho_l = \frac{\text{cov}(r_t, r_{t-l})}{\text{var}(r_t)} = \frac{\gamma_l}{\gamma_0}$$

$$\rho_l = \frac{E(r_t - \mu)(r_{t-l} - \mu)}{E(r_t - \mu)^2}$$

- Sequence $\rho_1, \rho_2, \rho_3, \ldots$ is autocorrelation function (ACF)
- Graphical representation of $\rho_1, \rho_2, \rho_3, \ldots$ is correlogram.

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Correlation coefficient: reminder

- $\rho(X,Y) = \frac{\text{cov}(X,Y)}{\sqrt{var(X)var(Y)}}$
- $-1 \leq \rho(X,Y) \leq 1$
- $\rho(X,Y) = \rho(Y,X)$
- $X = r_t, \ \ Y = r_{t-l}$
- $\rho_l = \frac{\text{cov}(r_t, r_{t-l})}{\sqrt{\text{var}(r_t)\ \text{var}(r_{t-l})}}$

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Autocorrelation function (ordinary) II

- Properties:

1. $\gamma_0 = \text{var}(r_t) \Rightarrow \rho_0 = 1$
2. $\gamma_l = \gamma_{-l} \Rightarrow \rho_l = \rho_{-l}$
3. $|\rho_l| \leq 1, \ \ l = 0,1,2, \ldots$
4. *Stationary time series is not serially correlated if and only if* $\rho_l = 0$, for all positive *l*.

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Sample autocorrelation function

$$\rho_l = \frac{E(r_t-\mu)(r_{t-l}-\mu)}{E(r_t-\mu)^2}$$

$Time$ series of lenght T:

$r_1, r_2, \ldots, r_T, \bar{r} - sample\ mean$

The lag-l sample autocorrelation coefficient

$$\hat{\rho}_l = \frac{\sum_{t=l+1}^{T}(r_t-\bar{r})\,(r_{t-l}-\bar{r})}{\sum_{t=1}^{T}(r_t-\bar{r})^2}, l = 0, \ldots, T-2$$

$Sequence$: $\hat{\rho}_1, \hat{\rho}_2, \ldots$ is sample autocorrelation function ($SACF$)

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Sample autocorrelation function II

If $\mathrm{r_t}$ *is iid* sequence (independent and identically distributed)of random variables, with constant variance, *then*:

1. $\hat{\rho}_l$ − biased, but consistent estimator

2. var($\hat{\rho}_l$)= $\frac{1}{\mathrm{T}}$

3. For **large** samples: $\hat{\rho}_l$: $AN\left(0, \frac{1}{T}\right)$

$$\Rightarrow z = \frac{\hat{\rho}_l - 0}{\sqrt{\frac{1}{\mathrm{T}}}} = \hat{\rho}_l\sqrt{T}: N(0,1)$$

$\Rightarrow \mathrm{P}\left[-1.96 \le \hat{\rho}_l\sqrt{T} \le 1.96\right] = 0.95$

$\Rightarrow \mathrm{P}\left[-1.96/\sqrt{T} \le \hat{\rho}_l \le 1.96/\sqrt{T}\right] = 0.95$

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Sample autocorrelation function III

If r_t *is* weakly stationary time series, *then*:

1. $\hat{\rho}_l -$ biased, but consistent estimator

2. $\text{var}(\hat{\rho}_l) = \frac{1}{T}\left(1 + 2\hat{\rho}_1^2 + 2\hat{\rho}_2^2 + \cdots + 2\hat{\rho}_{l-1}^2\right)$

3. For **large** samples and r_t being Gaussian process ($\rho_j = 0, j > l$)

$$\hat{\rho}_l : \ AN(0, var(\hat{\rho}_l))$$

$$\Rightarrow z = \frac{\hat{\rho}_l - 0}{\sqrt{(1+2\hat{\rho}_1^2+2\hat{\rho}_2^2+\cdots+2\hat{\rho}_{l-1}^2)/T}} : N(0,1)$$

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Autocorrelation tests

1. **Is there autocorrelation at certain lag *l*?**

   H_0: $\rho_l = 0$, H_1: $\rho_l \neq 0$

2. **Is there joint autocorrelation up to order *m*?**

   H_0: $\rho_1 = \rho_2 = \ldots = \rho_m = 0$,

   H_1: At least one autocorrelation coefficient (out of the first *m*) is not equal to zero.

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Is there autocorrelation at certain lag *l*? (H_0: $\rho_l = 0$)

- The hypothesis H_0: $\rho_l = 0$ is tested against the alternative H_1: $\rho_l \neq 0$ by checking whether estimator of the lag-*l* autocorrelation coefficient is an element of the interval $[-1.96/\sqrt{T}, 1.96/\sqrt{T}]$

- Null hypothesis cannot be rejected if:

$$\hat{\rho}_l \in [-1.96/\sqrt{T}, 1.96/\sqrt{T}]$$

- Null hypothesis is rejected at the 5% significance level if

$$\hat{\rho}_l \notin [-1.96/\sqrt{T}, 1.96/\sqrt{T}]$$

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Is there joint autocorrelation up to order *m*? (H_0: $\rho_1 = \rho_2 = \ldots = \rho_m = 0$)

- Box-Pierce and Box-Ljung test statistics are applied:

$$Q^*(m) = BP(m) = T\sum_{l=1}^{m} \hat{\rho}_l^{\,2} : \chi^2_m$$

$$Q(m) = BLj(m) = T(T+2)\sum_{l=1}^{m} \frac{\hat{\rho}_l^{\,2}}{T-l} : \chi^2_m$$

$$m = \sqrt{T},\ T/4,\ \ln(T)$$

- Null hypothesis is rejected at the 5% significance level if *Q(m)* is greater than the appropriate 5% critical value of chi-squared distribution with *m* degrees of freedom.

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Distribution of BP and BLj

- $BP(m) = T\sum_{l=1}^{m}\hat{\rho}_l^2.$
- If the null is true:

$$\hat{\rho}_1 : N(0,1/T) \Rightarrow z_1 = \sqrt{T}\hat{\rho}_1 : N(0,1)$$
$$\hat{\rho}_2 : N(0,1/T) \Rightarrow z_2 = \sqrt{T}\hat{\rho}_2 : N(0,1)$$
$$\ldots$$
$$\hat{\rho}_m : N(0,1/T) \Rightarrow z_m = \sqrt{T}\hat{\rho}_m : N(0,1)$$

$$\underbrace{z_1^2 + z_2^2 + \ldots + z_m^2}_{\Downarrow} : \chi_m^2$$
$$T\sum_{l=1}^{m}\hat{\rho}_l^2 = BP(m) : \chi_m^2.$$

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Examples: The use of autocorrelation function

1. Numerical example on testing if time series is white noise
2. Example of white noise generated within EVIEWS.
3. Numerical example on calculating Box-Ljung statistic

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Example 1

- Based on time series data set of 164 observations with null mean and constant variance, sample autocorrelation coefficients are calculated for lags 1, 2,..., 10:

$\hat{\rho}_1$	$\hat{\rho}_2$	$\hat{\rho}_3$	$\hat{\rho}_4$	$\hat{\rho}_5$	$\hat{\rho}_6$	$\hat{\rho}_7$	$\hat{\rho}_8$	$\hat{\rho}_9$	$\hat{\rho}_{10}$
-0.009	0.456	-0.069	-0.040	-0.073	-0.049	-0.062	-0.059	0.045	-0.038

- Can we consider this time series to be white noise?

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Example 1 (II)

- We need to check the validity of H_0: $\rho_l = 0$ against the alternative H_1: $\rho_l \neq 0$, $l=1,2,...,10$.
- If the null cannot be rejected for all ten lags, then there is no significant autocorrelation. Hence, the white noise seems to be appropriate description of this time series.
- The corresponding confidence interval with 95% probability is

$$[-0.153; 0.153]$$

- Time series of interest is not white noise:

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$$\hat{\rho}_2 = 0.456 \notin [-0.153; 0.153]$$

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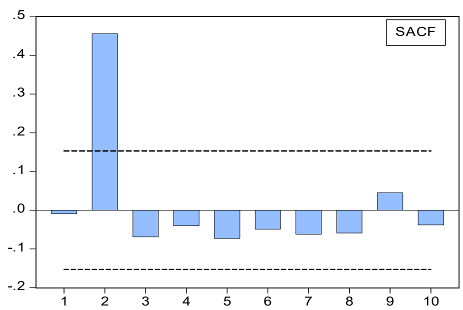
Example 1 (III)

- The graph of sample autocorrelation function (sample correlogram-SACF) gives an automatic answer to the question.
- Note: Dotted lines represent the 95% significance band $[-0.153; 0.153]$



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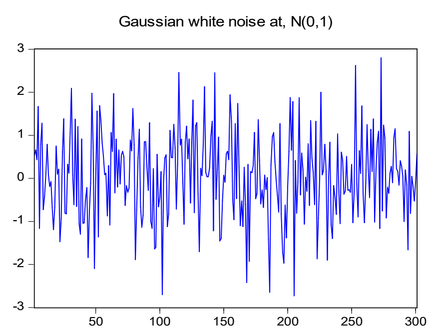
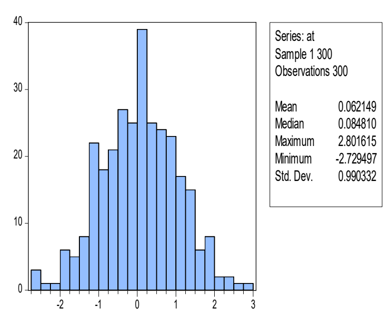
Example 2
Gaussian N(0,1) white noise:
data plot and histogram





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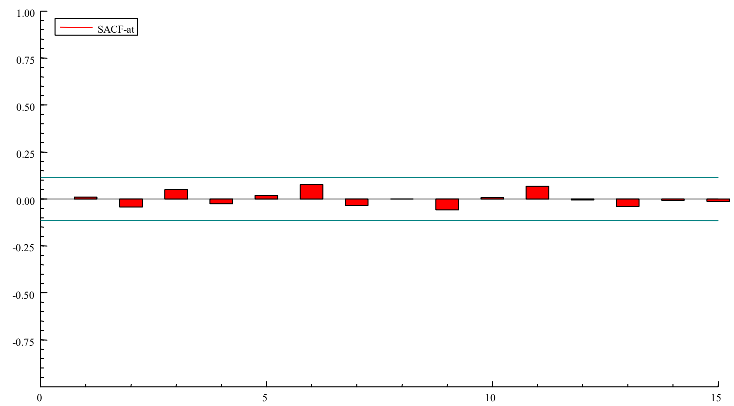
Example 2 (II)
Gaussian white noise: sample correlogram
(the 95% significance band:[-0.11,0.11])



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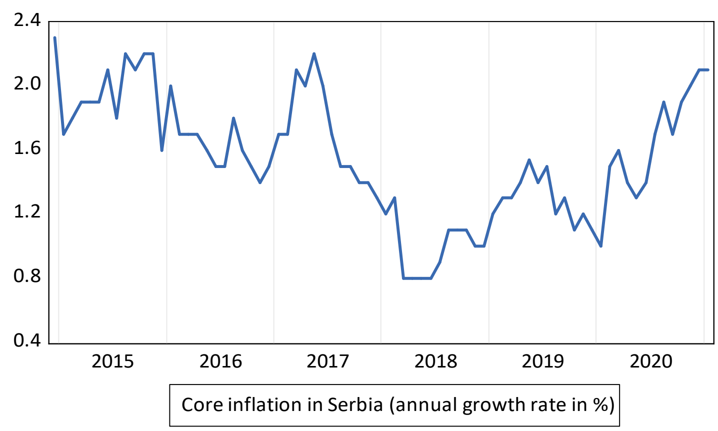
Example 3
Annual core inflation in Serbia, 2014:12-2021:1 (T=74)
Analysis based on SACF and BLj test
(core_inflation.wf1)



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Example 3 (II)

Analysis of sample autocorrelation function for core inflation in Serbia, 2014:12-2021:1 (*T*=74)

Lag	SACF	Is the autocorrelation significant?
1	0.824	YES
2	0.754	YES
3	0.657	YES
4	0.564	YES
5	0.508	YES
6	0.402	YES
7	0.327	YES
8	0.287	YES

- The 95% significance band: [-0.23;0.23]

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Example 3 (III)

Testing for joint autocorrelation of core inflation in Serbia, 2014:12-2021:1 (*T*=74)

$H_0: \rho_1 = \rho_2 = \dots = \rho_m = 0,\ H_1:\ H_0$ is not true

$$BLj(m) = Q(m) = T(T+2)\sum_{l=1}^{m}\frac{\hat{\rho}_i^{\,2}}{T-l} : \chi^2_m$$

$H_0:\ \rho_1 = \rho_2 = \dots = \rho_8 = 0, H_1: H_0$ is not true

$$Q(8) = 74 * 76$$

$$* \left[\frac{(0.824)^2}{(74-1)} + \frac{(0.754)^2}{(74-2)} + \frac{(0.657)^2}{(74-3)} + \frac{(0.564)^2}{(74-4)} + \frac{(0.508)^2}{(74-5)} + \frac{(0.402)^2}{(74-6)} + \frac{(0.327)^2}{(74-7)} + \frac{(0.287)^2}{(74-8)}\right]$$

$Q(8) = 206.9 > \chi^2_8(0.05) = 15.51 \Rightarrow H_0$ is rejected.

Time series exhibits significant joint autocorrelation of order 8.

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Example 3 (Eviews output of SACF)

Sample: 2014M12 2021M01
Included observations: 74

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.824	0.824	52.352	0.000
		2	0.754	0.232	96.731	0.000
		3	0.657	-0.037	130.94	0.000
		4	0.564	-0.064	156.52	0.000
		5	0.508	0.062	177.58	0.000
		6	0.402	-0.144	190.92	0.000
		7	0.327	-0.044	199.87	0.000
		8	0.287	0.102	206.87	0.000

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