1. Dat je sistem simultanih jednačina:

$P_t = \alpha_0 + \alpha_1 W_t + \varepsilon_{1t}$
$W_t = \beta_0 + \beta_1 P_t + \beta_2 U_t + \varepsilon_{2t}$

(P_t- cene, W_t- plata i U_t- nezaposlenost)

a) Odrediti endogene i egzogene promenljive u datom sistemu.
b) Šta je strukturna, a šta redukovana forma ovog sistema?

Rešenje:

a) Endogene promenljive su: P_t i W_t.
Egzogene promenljive su: 1 i U_t.

b) Polazni model predstavlja strukturnu formu modela u okviru koje se kao objašnjavajuće promenljive mogu javiti i endogene i egzogene promenljive. Za razliku od strukturne forme, u redukovanoj formi svaka endogena promenljiva je funkcija isključivo egzogenih promenljivih (za dinamičke modele, kakvi su svi zadaci u nastavku, i pomaknutih endogenih).

Redukovane forme su:

$$P_t = \alpha_0 + \alpha_1 W_t + \varepsilon_{1t} = \alpha_0 + \alpha_1(\beta_0 + \beta_1 P_t + \beta_2 U_t + \varepsilon_{2t}) + \varepsilon_{1t}$$

$$(1 - \alpha_1\beta_1)P_t = \alpha_0 + \alpha_1\beta_0 + \alpha_1\beta_2 U_t + \varepsilon_{1t} + \alpha_1\varepsilon_{2t}$$

$$\boldsymbol{P_t = \frac{\alpha_0 + \alpha_1\beta_0}{1 - \alpha_1\beta_1} + \frac{\alpha_1\beta_2}{1 - \alpha_1\beta_1} U_t + \frac{\varepsilon_{1t} + \alpha_1\varepsilon_{2t}}{1 - \alpha_1\beta_1}}$$

$$W_t = \beta_0 + \beta_1 P_t + \beta_2 U_t + \varepsilon_{2t}$$

$$W_t = \beta_0 + \beta_1\left(\frac{\alpha_0 + \alpha_1\beta_0}{1 - \alpha_1\beta_1} + \frac{\alpha_1\beta_2}{1 - \alpha_1\beta_1} U_t + \frac{\varepsilon_{1t} + \alpha_1\varepsilon_{2t}}{1 - \alpha_1\beta_1}\right) + \beta_2 U_t + \varepsilon_{2t}$$

$$\boldsymbol{W_t = \frac{\beta_0 + \alpha_0\beta_1}{1 - \alpha_1\beta_1} + \frac{\beta_2}{1 - \alpha_1\beta_1} U_t + \frac{\beta_1\varepsilon_{1t} + \varepsilon_{2t}}{1 - \alpha_1\beta_1}}$$

2. Posmatramo sistem simultanih jednačina:

$Y_t = \alpha_0 + \alpha_1 Y_{t-1} + \alpha_2 I_t + \alpha_3 t + \varepsilon_{1t}$
$I_t = \beta_0 + \beta_1 Y_t + \varepsilon_{2t}$

(Y_t- proizvodnja, t- linearni trend i I_t-investicije)

a) Odrediti endogene i predeterminisane promenljive u datom sistemu.
b) Odrediti redukovanu formu modela.
c) Objasniti zašto primena metoda ONK nije podobna prilikom ocenjivanja datog strukturnog modela.

Rešenje:

a) Endogene promenljive su: Y_t i I_t.
Predeterminisane promenljive su: 1, Y_{t-1} i t.

b)

Redukovane forme su:

$$Y_t = \alpha_0 + \alpha_1 Y_{t-1} + \alpha_2 I_t + \alpha_3 t + \varepsilon_{1t} = \alpha_0 + \alpha_1 Y_{t-1} + \alpha_2(\beta_0 + \beta_1 Y_t + \varepsilon_{2t}) + \alpha_3 t + \varepsilon_{1t}$$

$$(1 - \alpha_2\beta_1)Y_t = \alpha_0 + \alpha_2\beta_0 + \alpha_1 Y_{t-1} + \alpha_3 t + \varepsilon_{1t} + \alpha_2\varepsilon_{2t}$$

$$\boldsymbol{Y_t = \frac{\alpha_0 + \alpha_2\beta_0}{1 - \alpha_2\beta_1} + \frac{\alpha_1}{1 - \alpha_2\beta_1} Y_{t-1} + \frac{\alpha_3}{1 - \alpha_2\beta_1} t + \frac{\varepsilon_{1t} + \alpha_2\varepsilon_{2t}}{1 - \alpha_2\beta_1}}$$

$$I_t = \beta_0 + \beta_1 Y_t + \varepsilon_{2t}$$

$$I_t = \beta_0 + \beta_1\left(\frac{\alpha_0 + \alpha_2\beta_0}{1 - \alpha_2\beta_1} + \frac{\alpha_1}{1 - \alpha_2\beta_1} Y_{t-1} + \frac{\alpha_3}{1 - \alpha_2\beta_1} t + \frac{\varepsilon_{1t} + \alpha_2\varepsilon_{2t}}{1 - \alpha_2\beta_1} \right) + \varepsilon_{2t}$$

$$\boldsymbol{I_t = \frac{\beta_0 + \alpha_0\beta_1}{1 - \alpha_2\beta_1} + \frac{\alpha_1\beta_1}{1 - \alpha_2\beta_1} Y_{t-1} + \frac{\alpha_3\beta_1}{1 - \alpha_2\beta_1} t + \frac{\beta_1\varepsilon_{1t} + \varepsilon_{2t}}{1 - \alpha_2\beta_1}}$$

c) Jedna od pretpostavki linearnog regresionog modela: Objašnjavajuće promenljive nisu slučajne promenljive, već uzimaju fiksirane vrednosti iz ponovljenih uzoraka (nisu korelisane sa slučajnom greškom).

Ako posmatramo prvu jednačinu modela jedna od objašnjavajućih promenljivih je I_t. Na osnovu redukovane forme za investicije vidimo da je promenljiva I_t linearna funkcija slučajne greške ε_{1t} (pokazati!). Primena metoda ONK na prvu jednačinu daće pristrasne ocene usled narušene gore pomenute pretpostavke.

Ako posmatramo drugu jednačinu modela jedna od objašnjavajućih promenljivih je Y_t. Na osnovu redukovane forme za proizvodnju vidimo da je promenljiva Y_t linearna funkcija slučajne greške ε_{2t} (pokazati!). Primena metoda ONK na drugu jednačinu daće takođe pristrasne ocene usled narušene gore pomenute pretpostavke.

3. Dat je sistem simultanih jednačina:

$$P_t = \alpha_0 + \alpha_1 W_t + \alpha_2 P_{t-1} + \varepsilon_{1t}$$
$$W_t = \beta_0 + \beta_1 P_t + \beta_2 M_t + \varepsilon_{2t}$$

(P_t- cene, W_t- plate i M_t-novčana masa)

Ocenjene su redukovane forme:

$$\widehat{P_t} = 34.43 + 0.81\mathrm{M_t} + 0.67\mathrm{P}_{t-1}$$

$$\widehat{W_t} = 69.88 + 0.57\mathrm{M_t} + 0.84\mathrm{P}_{t-1}$$

Oceniti parametre nagiba strukturne forme metodom INK.

Rešenje:

Endogene promenljive su: P_t i W_t.
Predeterminisane promenljive su: 1, M_t i P_{t-1}.

Ocene parametara strkuturne forme primenom metoda INK dobijaju se na osnovu njihove veze sa parametrima redukovane forme.

Redukovane forme:

$$P_t = \alpha_0 + \alpha_1 W_t + \alpha_2 P_{t-1} + \varepsilon_{1t} = \alpha_0 + \alpha_1(\beta_0 + \beta_1 P_t + \beta_2 M_t + \varepsilon_{2t}) + \alpha_2 P_{t-1} + \varepsilon_{1t}$$

$$(1 - \alpha_1\beta_1)P_t = \alpha_0 + \alpha_1\beta_0 + \alpha_1\beta_2 M_t + \alpha_2 P_{t-1} + \varepsilon_{1t} + \alpha_1\varepsilon_{2t}$$

$$\boldsymbol{P_t = \frac{\alpha_0+\alpha_1\beta_0}{1-\alpha_1\beta_1} + \frac{\alpha_1\beta_2}{1-\alpha_1\beta_1} M_t + \frac{\alpha_2}{1-\alpha_1\beta_1} P_{t-1} + \frac{\varepsilon_{1t}+\alpha_1\varepsilon_{2t}}{1-\alpha_1\beta_1}}$$

$$W_t = \beta_0 + \beta_1 P_t + \beta_2 M_t + \varepsilon_{2t}$$

$$W_t = \beta_0 + \beta_1\left(\frac{\alpha_0+\alpha_1\beta_0}{1-\alpha_1\beta_1} + \frac{\alpha_1\beta_2}{1-\alpha_1\beta_1} M_t + \frac{\alpha_2}{1-\alpha_1\beta_1} P_{t-1} + \frac{\varepsilon_{1t}+\alpha_1\varepsilon_{2t}}{1-\alpha_1\beta_1} \right) + \beta_2 M_t + \varepsilon_{2t}$$

$$\boldsymbol{W_t = \frac{\beta_0+\alpha_0\beta_1}{1-\alpha_1\beta_1} + \frac{\beta_2}{1-\alpha_1\beta_1} M_t + \frac{\alpha_2\beta_1}{1-\alpha_1\beta_1} P_{t-1} + \frac{\beta_1\varepsilon_{1t}+\varepsilon_{2t}}{1-\alpha_1\beta_1}}$$

Odnosno:

$$P_t = c_0 + c_1 M_t + c_2 P_{t-1} + v_{1t}$$

$$W_t = d_0 + d_1 M_t + d_2 P_{t-1} + v_{2t}$$

Gde je:

$$c_0 = \frac{\alpha_0 + \alpha_1\beta_0}{1-\alpha_1\beta_1}, \quad c_1 = \frac{\alpha_1\beta_2}{1-\alpha_1\beta_1}, \quad c_2 = \frac{\alpha_2}{1-\alpha_1\beta_1},$$

$$d_0 = \frac{\beta_0 + \alpha_0\beta_1}{1-\alpha_1\beta_1}, \quad d_1 = \frac{\beta_2}{1-\alpha_1\beta_1}, \quad d_2 = \frac{\alpha_2\beta_1}{1-\alpha_1\beta_1}.$$

U zadatku su nam date ocene redukovane forme:

$$\widehat{c_1} = 0.81, \quad \widehat{c_2} = 0.67,$$

$$\widehat{d_1} = 0.57, \quad \widehat{d_2} = 0.84.$$

Važi sledeće:

$$\alpha_1 = \frac{\alpha_1\beta_2}{1-\alpha_1\beta_1} : \frac{\beta_2}{1-\alpha_1\beta_1} = \frac{c_1}{d_1}$$

$$\boldsymbol{\widehat{\alpha_1} = \frac{\widehat{c_1}}{\widehat{d_1}} = \frac{0.81}{0.57} = 1.421}$$

$$\beta_1 = \frac{\alpha_2\beta_1}{1-\alpha_1\beta_1} : \frac{\alpha_2}{1-\alpha_1\beta_1} = \frac{d_2}{c_2}$$

$$\boldsymbol{\widehat{\beta_1} = \frac{\widehat{d_2}}{\widehat{c_2}} = \frac{0.84}{0.67} = 1.254}$$

$$c_1 = \frac{\alpha_1\beta_2}{1-\alpha_1\beta_1} \quad \rightarrow \quad \beta_2 = \frac{c_1(1-\alpha_1\beta_1)}{\alpha_1}$$

$$\boldsymbol{\widehat{\beta_2} = \frac{\widehat{c_1}(1-\widehat{\alpha_1}\widehat{\beta_1})}{\widehat{\alpha_1}} = -0.446}$$

$$d_2 = \frac{\alpha_2\beta_1}{1-\alpha_1\beta_1} \quad \rightarrow \quad \alpha_2 = \frac{d_2(1-\alpha_1\beta_1)}{\beta_1}$$

$$\boldsymbol{\widehat{\alpha_2} = \frac{\widehat{d_2}(1-\widehat{\alpha_1}\widehat{\beta_1})}{\widehat{\beta_1}} = -0.524}$$

4. Posmatramo jednostavni agregatni model:

$$C_t = \alpha_0 + \alpha_1 Y_t + \epsilon_{1t}$$
$$I_t = \beta_0 + \beta_1 Y_{t-1} + \beta_2 Y_t + \epsilon_{2t}$$
$$Y_t = C_t + I_t$$

(C_t- potrošnja, Y_t- dohodak i I_t-investicije)

Podaci dobijeni na osnovu uzorka od 39 opservacija su:

$$\sum c_t y_{t-1} = 16.5, \sum y_{t-1}^2 = 29.75, \sum i_t y_{t-1} = 22.37, \sum y_t y_{t-1} = 19.5,$$

$$\overline{C_t} = 21.7, \overline{Y_t} = 50.1\ \overline{Y_{t-1}} = 42, \overline{I_t} = 23.52.$$

a) Oceniti redukovane forme modela.
b) Oceniti parametar α_1 u jednačini potrošnje metodom INK.

Rešenje:

a)

Endogene promenljive su: C_t , I_t i Y_t.

Predeterminisane promenljive su: 1 i Y_{t-1}.

Redukovane forme su:

$$C_t = c_0 + c_1 Y_{t-1} + v_{1t}$$
$$I_t = d_0 + d_1 Y_{t-1} + v_{2t}$$

$$Y_t = e_0 + e_1 Y_{t-1} + v_{3t}$$

$$\widehat{c_1} = \frac{\sum c_t y_{t-1}}{\sum y_{t-1}^2} = \frac{16.5}{29.75} = 0.555$$

$$\widehat{c_0} = \overline{C_t} - \widehat{c_1} * \overline{Y_{t-1}} = 21.7 - 0.555 * 42 = -1.61$$

$$\widehat{d_1} = \frac{\sum i_t y_{t-1}}{\sum y_{t-1}^2} = \frac{22.37}{29.75} = 0.752$$

$$\widehat{d_0} = \overline{I_t} - \widehat{d_1} * \overline{Y_{t-1}} = 23.52 - 0.752 * 42 = -8.064$$

$$\widehat{e_1} = \frac{\sum y_t y_{t-1}}{\sum y_{t-1}^2} = \frac{19.5}{29.75} = 0.655$$

$$\widehat{e_0} = \overline{Y_t} - \widehat{e_1} * \overline{Y_{t-1}} = 50.1 - 0.655 * 42 = 22.59$$

Ocenjene redukovane forme su:

$$\boldsymbol{\widehat{C_t} = -1.61 + 0.555 Y_{t-1}}$$

$$\boldsymbol{\widehat{I_t} = -8.064 + 0.752 Y_{t-1}}$$

$$\boldsymbol{\widehat{Y_t} = 22.59 + 0.655 Y_{t-1}}$$

b)

Na osnovu parametara strukturne forme dobijaju se sledeće redukovane forme (pokazati):

$$C_t = \frac{\alpha_0(1-\beta_2)+\alpha_1\beta_0}{1-\alpha_1-\beta_2} + \frac{\alpha_1\beta_1}{1-\alpha_1-\beta_2} Y_{t-1} + v_{1t}$$

$$I_t = \frac{\beta_0(1-\alpha_1)+\alpha_0\beta_2}{1-\alpha_1-\beta_2} + \frac{\beta_1(1-\alpha_1)}{1-\alpha_1-\beta_2} Y_{t-1} + v_{2t}$$

$$Y_t = \frac{\alpha_0+\beta_0}{1-\alpha_1-\beta_2} + \frac{\beta_1}{1-\alpha_1-\beta_2} Y_{t-1} + v_{3t}$$

$$\alpha_1 = \frac{\alpha_1\beta_1}{1-\alpha_1-\beta_1} : \frac{\beta_1}{1-\alpha_1-\beta_1} = \frac{c_1}{e_1}$$

$$\widehat{\boldsymbol{\alpha_1}} = \frac{\widehat{\boldsymbol{c_1}}}{\widehat{\boldsymbol{e_1}}} = \frac{\mathbf{0.555}}{\mathbf{0.655}} = \mathbf{0.85}$$