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Cagan's paradox and money demand in hyperinflation: Revisited at daily frequency

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A B S T R A C T

Using daily data the Cagan money demand is estimated and accepted for the most severe portion of Serbia's 1992–1993 hyperinflation, i.e. its last 6 months. An implication is that the public adjusted daily throughout this extreme period. Moreover, the obtained semi-elasticity estimates are by far lower than those previously found using monthly data sets. Consequently, the daily estimates reject the longstanding Cagan's paradox, based on monthly studies, by showing that the economy has been on the correct, increasing side of the Laffer curve almost through the end of hyperinflation. This strongly supports the view that hyperinflation is triggered and driven all way through its end by the government's hunt for non-decreasing seigniorage. Daily adjustments of public in hyperinflation can account for the difference between the results obtained at daily and monthly frequencies, calling into question the latter. Some evidence is offered that the findings of this paper may hold for other hyperinflations.

1. Introduction

There has been a longstanding interest, initiated by Cagan (1956) in exploring money demand in hyperinflation, and most relevant studies have been done using monthly data. However, hyperinflations are extreme events with daily inflation rates comparable to quarterly or annual rates in moderate inflation economies. Thus a conjecture is that in hyperinflation the public adjusts at daily frequency and hence that monthly observations could offer a misleading picture of the public's behavior (Taylor,

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2001). In addition, hyperinflations are short-lived episodes characteristically lasting around 20 months. Obviously this is very small sample for sound estimation, and the problem has typically been moderated by extending it to include the lower inflation period preceding hyperinflation. The latter practice leads us to another problem with monthly estimates which is that they encompass a non-homogenous period. The issue is further complicated by some evidence that even hyperinflations themselves are non-homogenous events, each with its own distinct severe portion (Michael et al., 1994). Again the latter proposition cannot be demonstrated and taken care of with monthly observations. All of the above suggest that previous monthly studies of money demand could be misleading, and that daily data is required to bring to light agents' behavior in periods of hyperinflation.

This paper examines the money demand schedule at daily frequency in an advanced stage of hyperinflation, and contrasts it with monthly studies. In particular we ask whether money demand estimates at daily frequency can resolve the longstanding Cagan's paradox derived from monthly studies.

Cagan's (1956) paradox states that in hyperinflation authorities tend to expand money supply at a rate well beyond that which would maximize their inflation tax revenue. As early as the early 1970s Barro (1972) reported widely different estimates of revenue-maximizing rates: Friedman (1971) came out with the maximizing inflation rate below 20% per year, Cagan (1956) with around 20% per month, and Barro (1972) with 140% per month.

Mainstream research has followed Cagan's model, generating estimates that uniformly reinforce the paradox of non-optimal seigniorage from excessive money creation (Michael et al., 1994). Specifically, highly efficient estimates of the Cagan money demand that hold for a wide set of expectation formation processes supported Cagan's results (Taylor, 1991; Engsted, 1994). The estimated semi-elasticity varies in ranges similar to those found in Cagan (1956), i.e. from 3 to 6, and their inverse values provide revenue-maximizing inflation rates in the range of 17–33% per month. Statistical tests confirm that average inflation rates across hyperinflations significantly exceed the seigniorage maximizing ones. This suggests that for a substantial portion of each hyperinflation, economies were placed on the wrong, decreasing side of the inflation tax Laffer curve.

Some alternative research however has suggested that in the extreme portion of hyperinflation semi-elasticity might decrease, placing an economy on the correct side of the Laffer curve for a large part of hyperinflation. Thus Michael et al. (1994) focused on the most extreme period of the German hyperinflation, including the final months that have been previously considered as outliers, and obtained a seven times smaller semi-elasticity than reported above, which goes a long way to resolving Cagan's paradox. The result also suggests that the considered extreme period represents a distinct portion of the German hyperinflation. However, a serious shortcoming of the Michael et al. (1994) result is its reliance on a brutally small monthly sample of only 14–16 observations, which severely limits its robustness.

Another strand of research abandons Cagan's framework and opts for money demand schedules that allow for money substitutes, where semi-elasticity decreases (elasticity increases) as inflation accelerates. Advancing that line, Barro (1970, 1972) obtained estimates for the five classical hyperinflations showing that in three cases governments were on the increasing segment of the Laffer curve through to the end of hyperinflation. However, in the most extreme episodes of Germany and Hungary II (1945–1946) the economies were on the wrong side of the curve for the last 3 and 4 months of hyperinflation, respectively.

This paper explores the most severe portion of the Serbian hyperinflation of 1992–1993, at daily frequency. As opposed to previous monthly studies, including Michael et al. (1994), we are able to rely on a large sample of daily data covering the last 6–7 months of extreme hyperinflation. The Serbian hyperinflation itself is an extreme event, second only (in the 20th century) to Hungary II in extremity and to 1920s Russian in duration (Petrović et al., 1999). The severe portion that we shall examine is characterized by an average monthly currency depreciation rate of 10,700% which is 33 times higher than the average inflation rate (322%) in the German hyperinflation. Our motivation for choosing this period is a conjecture that the public adjusts daily in these extreme conditions.

The paper proceeds as follows. Section 2 gives a background of the Serbian hyperinflation while demonstrating its severe nature in the period to be explored and describes the evolution of the series, particularly inflation tax and real money balances. It also explains the data set that is used. Section 3

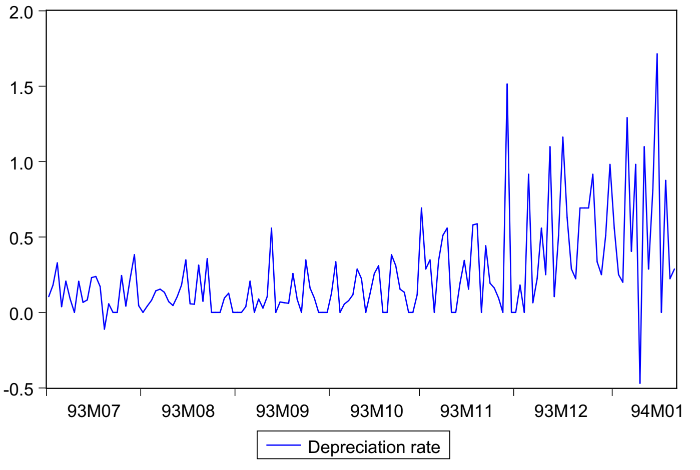

Fig. 1. Exchange rate depreciation (Δe). Exchange rate represents domestic currency per one German mark.

examines whether the Cagan money demand holds for the last 6 months of extreme hyperinflation. In Section 4, Cagan's paradox is revisited at daily frequency using the obtained money demand semi-elasticity estimates; the latter are compared to semi-elasticity estimates previously attained at monthly frequency. Conclusions are offered in Section 5.

2. The extreme portion of Serbian hyperinflation: background

2.1. The data set

The Serbian hyperinflation is, as explained above, one of the most extreme events in the twentieth century. It started in February 1992, when monthly inflation exceeded the conventional 50% rate, then accelerated significantly during 1993 and was eventually halted by end-January 1994 (Petrović et al., 1999). We shall explore the most severe sub-period of the hyperinflation that runs from July 1993 through to its end, employing daily data on money supply and exchange rate.

The data to be used are black market exchange rates and the currency in circulation (cash) as money supply, both with a daily frequency. These series are relatively sound compared to other data in hyperinflation. Data for exchange rates were generated by the black market and recorded daily, as opposed to price indices that were calculated magnitudes based on monthly surveys and hence distorted in hyperinflation. Exchange rates determined in a free market are subject to much lower measurement error than are price indices. Therefore in order to address the errors-in-variable problem one should use the former in place of the latter, even in monthly studies where both are available (Petrović and Mladenović, 2000). At daily frequency, however, the price level data is not available.

The source for the daily cash series is the central bank of Serbia.[1] Again this series is relatively sound. Namely, the central bank, as the printer and distributor of cash, had direct control and evidence of cash expansion, and thus was able to record its magnitude quite accurately.[2]

[1] At the time of hyperinflation it was the central bank of FR Yugoslavia (Serbia and Montenegro) and both money supply and exchange rate correspond to FR Yugoslavia. However, since Montenegro accounts for only 5% of joint GDP, we are referring to this hyperinflation as the Serbian. Furthermore the central bank, which generated hyperinflation, was effectively under Serbian control.

[2] This was not, however, the case with M1 (Petrović et al., 1999).

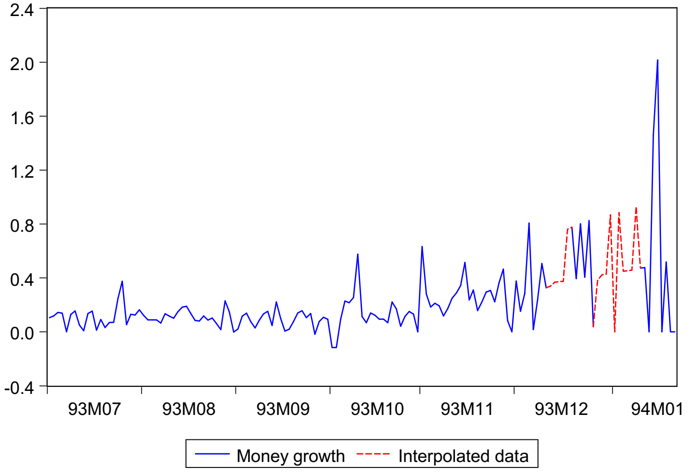

Fig. 2. Money growth (Δm).

An additional reason to opt for cash money is that the average share of cash in the base money was around 80%, representing the main source for seigniorage in the period considered. Finally, most of classical hyperinflations are also studied employing notes in circulations (Barro, 1970,1972).

The daily data for exchange rates is available for the whole period, i.e. through January 1994. However some observations on the cash money supply are missing in December and January, in particular for December 13–19, and December 28–January 10 periods. We have interpolated them (see Fig. 2),[3] and used this money series when analyzing the whole period as in Fig. 3. However, partly due to missing data, the sample used for estimation runs through December. Also, it covers five working days per week, since the money supply did not change over weekends.

2.2. *Money supply and exchange rate dynamics*

The daily evolution of exchange rate depreciation (Δe) and money growth (Δm) are shown in Figs. 1 and 2, and in Table 1. The exchange rate depreciation (Δe) and the money growth (Δm) doubled in July as compared to June thus attaining a new plateau through October. Subsequently, they accelerated significantly again in November, December and January. The same pattern is followed by variability of these series as shown in Figs. 1 and 2, and by corresponding standard deviations in Table 1. Econometric tests confirm the described pattern. Specifically, the daily series of Δm and Δe exhibited a structural break by the end of June increasing both in level and persistence, i.e. switching from stationary to non-stationary series.[4] The latter even formally points to July 1993 as the start of distinctive portion of hyperinflation, and motivates our choice of the period to be examined. Additionally, variability of the series sharply increases in December and January (cf.

[3] The exponential trend is used to interpolate logarithm of money supply (m). In addition, the level of money supply achieved on Friday is kept constant through Sunday, as there was no money creation over weekends.

[4] Various tests are employed all suggesting a break in persistence by the end of June, first in money growth and then in exchange rate depreciation. The null hypothesis that the series (Δm and Δe) are $I(1)$ throughout the sample against the alternative that the number of unit roots changes from 0 to 1 (Leybourne et al., 2003) is rejected in both cases. The test is based on the Elliot-Rothenberg-Stock type of DF t-ratio (Elliott et al., 1996). A two-step procedure (Busetti and Taylor, 2004) estimating the break point in level and then testing change in persistence confirmed that both occurred by the end of June. The results are available from the authors upon request.

Table 1
Daily exchange rate depreciation and money growth in Serbia.

Per day (%)	June 1993	July	August	September	October	November	December	January 1994
Δe								
Mean	4.1	9.2	7.7	7.2	11.6	21.5	36.4	40.6
St. dev.	4.7	9.5	7.9	8.7	13.5	28.3	33.1	44.8
Δm								
Mean	4.1	8.1	7.6	6.7	10.1	18.0	33.4	38.7
St. dev.	8.5	8.7	6.7	6.4	14.5	14.9	29.1	55.0

The rates above are continuous ones defined as log difference, e.g. $\Delta e = \ln(E/E_{-1})$. In December and January some observations for money supply are missing, and we have interpolated them (cf. Section 2.1). Data for January runs through Friday, January 21, since stabilization was introduced and hyperinflation halted on Monday, January 24.

Figs. 1 and 2, and Table 1), and the corresponding tests detect the break in their variability sometimes at the end of November or early December.[5]

To put this extreme period of the Serbian hyperinflation in comparative perspective one should look at monthly frequency. Thus, the monthly depreciation rate (Δe) varied from 216% in September to 1092% in December and 1218% in January. The corresponding discrete rates that are usually reported vary from 767% to 5,527,980% and 19,485,186%, respectively per month. For comparison, the average monthly discrete rate was 322% in the famous German hyperinflation versus 10,677% in the extreme portion (July–December) of the Serbia's hyperinflation.[6] In addition, the maximum monthly rate of monetary expansion (Δm) was 660% in Germany (November 1923), 3000% in Hungary (July 1946)[7] and 1002% in Serbia (December 1993). Therefore, we shall be exploring this distinctive and indeed extreme portion of the Serbian hyperinflation.

The severity of the period to be examined strongly suggests that the public adjusts their decisions daily rather than monthly. Both the level and the variability of currency depreciation and money growth at daily frequency (cf. Figs. 1 and 2, and Table 1) are large enough to imply daily adjustment in this extreme stage of hyperinflation. Thus, daily rates of currency depreciation and money growth are high even for the period from July through October, let alone for November and December. That is to say the scale of these rates would still be considerable even if they were to appear as quarterly or annual rates. Additionally, the standard deviations reported in Table 1 show that there is enough variability within the month at daily frequency, therefore strongly supporting our daily adjustments conjecture.

Some other hyperinflations, as reported in Table 2, have extreme portions that are comparable to the Serbian one. The average inflation and money growth rate for the last 6 months in the three reported hyperinflations strongly suggest that the public adjusts daily in their respective extreme periods. Therefore, the findings for Serbia at daily frequency obtained in this paper may well extend to at least the German, Greek and Hungarian (1945–1946) hyperinflations.

Monthly studies of money demand, apart from an attempt by Michael et al. (1994), have not been able to explain the last 3 months of the German hyperinflation (due to their severity), and hence routinely treat them as outliers. Nevertheless, this portion of the German hyperinflation is less severe than the corresponding three month (October–December) period in the Serbian episode, as the respective daily rates indicate: 14.8% and 16.4% in Germany vs. 23.2% and 20.5% in Serbia. Accordingly, explaining the extreme portion of Serbian hyperinflation at daily frequency would provide a framework for understanding the similar stage in the German case.

[5] The results are available from the authors upon request.

[6] The discrete rate is defined as $x = [(E/E_{-1}) - 1]$, while continues as $\Delta e = \ln(E/E_{-1})$, where E is the exchange rate. Comparing in terms of the latter, the German average inflation rate equals $\Delta p = 144\%$ vs. the extreme portion Serbian one $\Delta e = 470\%$. This still indicates that the latter is three times as severe as the former. However, as suggested by Cagan (1956), comparisons are regularly made in terms of discrete rates.

[7] For Germany and Hungary see Table 1, Barro (1972).

Table 2
Inflation and money growth in extreme portion of hyperinflation. Average for the last 6 months, expressed per day (%).

	Germany	Greece	Hungary II	Serbia
Inflation (Δp)	9.9 (14.8)	8.3	4.9	15.6 (23.2)
Money growth (Δm)	9.7 (16.4)	9.7	3.9	14.0 (20.5)

Average monthly inflation and money growth for the last 6 months are expressed per day by dividing monthly averages by 30. In the brackets are average rates for the last 3 months. Source: Cagan (1956) for Germany and Greece, and Barro (1972) for Hungary II. For Serbia average money growth and currency depreciation as a measure of inflation are derived from Table 1.

2.3. Seigniorage and inflation tax

Seigniorage is calculated as daily changes in cash money over exchange rate, and inflation tax as real cash holdings multiplied by depreciation rate. These two measures are equal only in a steady state. The results are depicted in Table 3.

The data reported in Table 3 suggests the existence of the Laffer curve property. Namely seigniorage and inflation tax are both relatively stable from August onwards, reaching the maximum in November and then declining in December and January. This development follows those of the exchange rate depreciation (Δe) and money growth (Δm) through November (cf. Table 1). July is an outlier with seigniorage and inflation tax almost as high as in November, but with money growth and exchange rate depreciation half of that in November. This result might be due to the sharp rise, i.e. doubling of money growth and exchange rate depreciation in July which perhaps had not been anticipated. Namely, these series (Δe and Δm) exhibit a structural break both in level and persistence by the end of June. Hence the observation for July could be off the Laffer curve.

With weekly observations, one finds that the maximum seigniorage is achieved in the last week of November (22–28), while the maximum for inflation tax is attained a week later, i.e. the beginning of December. Again, there is one outlier week in July.

Summarizing the trends explored above, we see that money growth and exchange rate depreciation rose abruptly in July 1993, remained relatively stable through October and then started to increase again. Accordingly, seigniorage and inflation tax were also initially stable, then increased and reached the maximum by the end of November 1993, and subsequently largely declined in December and January. This suggests that the economy was on the efficient, increasing side of the Laffer curve all way through November, and on the wrong side in December and January. The latter has been reflected in the movement of exchange rate depreciation (Δe) and money growth (Δm). Specifically, the daily data indicates that, after surpassing the maximum of the Laffer curve by the end of November, exchange rate depreciation (Δe) and money growth (Δm) surged and became considerably more volatile (cf. Table 1).

Thus the data suggests the existence of the inflation tax Laffer curve, and that the economy was on the efficient side of the curve throughout the severe hyperinflation, with the exception of the last 2 months (seven weeks). This evidence obtained at daily frequency for Serbia run against previous findings in monthly studies, while suggesting that governments carry on with money printing as long as non-decreasing seigniorage could be extracted. The latter challenges Cagan's paradox found at monthly frequency, and supports the view that seigniorage collection can explain both what triggers hyperinflation and how long it will last. However this evidence should and will be formally tested in what follows (cf. Section 3).

Table 3
Seigniorage and inflation tax in Serbia. Average per day, million German marks.

	June 1993	July	August	September	October	November	December 1–12 and 20–27	January 11–21
Seigniorage	2.4	3.4	3.1	2.6	2.9	3.5	2.9	1.5
Inflation tax	2.3	3.9	3.2	2.8	3.3	3.9	3.0	1.4

In December and January some observations for money supply are missing, hence we report sub-periods for which data is available.

2.4. Evolution of real money balances

Real money balances, defined as currency in circulation over exchange rate, or in logs, $(m - e)$, exhibit movement consistent with the Cagan money demand schedule.

As shown in Fig. 3, real money balances are relatively stable in the period from July through September 1993, when money growth (Δm) and exchange rate depreciation (Δe) are also stable (cf. Table 1). The sharp decrease in real money holdings during November and December coincides with the surge in hyperinflation. Thus developments of real money balances from July through December seem consistent with the Cagan money demand schedule. However, demonetization was halted in January 1994 despite extreme rates of money growth and currency depreciation. The latter may be due to the announcement of stabilization, first for the beginning of January and then postponed until the January 24 when it was enacted. Additionally half of the money data points for January are missing (see Fig. 2), and the interpolated observations used instead might be prone to measurement error. All this motivates us to skip January from the sample used to estimate money demand.

3. Estimating Cagan money demand

The Cagan money demand model we shall be looking at is:

$$m_t - e_t = -\alpha E_t(e_{t+1} - e_t) + u_t \tag{1}$$

The m_t and e_t are the natural logarithms of money and exchange rate, respectively, E_t is the conditional expectation operator, coefficient α is the semi-elasticity of money demand, and u_t is stationary velocity shock.

The model differs from the standard one in replacing the price level with the exchange rate. It can be thought of as the reduced form obtained by substituting out prices in the standard model using the purchasing power parity hypothesis.

Alternatively, one may argue that in hyperinflation the exchange rate behavior better reflects true inflation dynamics than do actual price movements. Beside an error-in-variables argument advanced above, one should invoke almost complete dollarization of an economy in hyperinflation. This means that practically all prices are set and most transactions performed in foreign currency, but also that the public expresses relevant magnitudes like real money, income, etc. in foreign money as well. Consequently economic agents look at the exchange rate movements rather than at those of prices while making their decisions in hyperinflation. This then suggests that exchange rate developments both

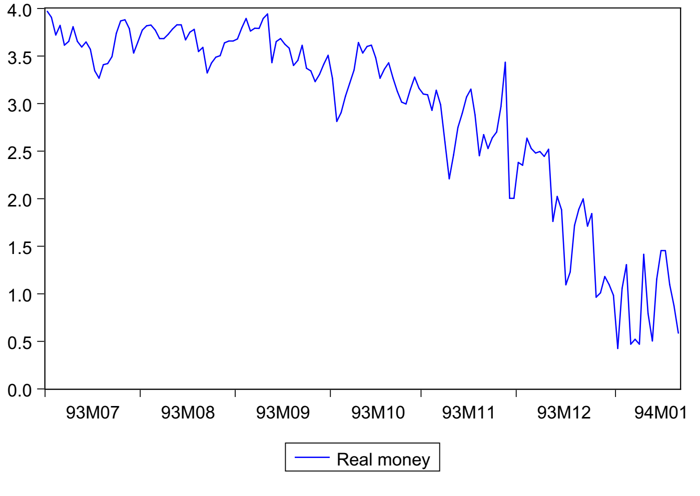

Fig. 3. Real money $(m - e)$.

Table 4
Unit root testing (period: July 1–December 31, 1993).

	e_t	m_t	$m_t - e_t$
Augmented Dickey-Fuller			
H_o: $I(3)$	−18.55	−15.89	
H_1: $I(2)$			
H_o: $I(2)$	−1.23	−1.03	−8.35
H_1: $I(1)$			
H_o: $I(1)$			−3.02
H_1: $I(0)$			
Kwiatkowski–Phillips–Schmidt–Shin			
H_o: $I(2)$	0.005	0.009	
H_1: $I(3)$			
H_o: $I(1)$	0.29	0.29	0.018
H_1: $I(2)$			
H_o: $I(0)$			1.78
H_1: $I(1)$			

The number of lags in ADF and KPSS tests is chosen as a minimum number of lags that eliminates autocorrelation. The number of corrections is equal to 9 for exchange rate and money, and 0 for real money. The unit root tests are based on the model with constant and trend with the 5% critical value for ADF test −3.45 (MacKinnon, 1991). The corresponding 5% critical value for the KPSS test is 0.15 (Kwiatkowski et al., 1992). The 5% critical value for the right tail of the ADF distribution is −0.90 in the model with constant and trend (Fuller, 1976).

influence and reflect the public's behavior, and hence should be a better proxy for expected inflation than the actual inflation rate. Moreover, one may also argue that the exchange rate is determined directly in the money market and not through relative price levels, and the Cagan model above captures this feature. In fact there is already some empirical evidence on the German (Engsted, 1996) and the Serbian (Petrović and Mladenović, 2000) hyperinflations that support the latter hypothesis. Additionally, some support is found even in a panel of 19 low-inflation developed economies (Mark and Sul, 2001).[8]

In monthly studies of high and hyperinflation (e.g. Taylor, 1991; Phylaktis and Taylor, 1993; Engsted, 1994; Petrović and Vujošević, 1996) it has been well documented that real money balances cointegrate with inflation rate, hence supporting the Cagan money demand. Moreover, the existence of this cointegration implies a super-consistent estimate of money demand semi-elasticity that holds for a wide array of expectations formation schemes. We shall now explore whether the same pattern emerges at daily frequencies in the severe portion of hyperinflation. Interestingly enough, the cointegration between real money balances and inflation rate was not found in a rare hyperinflation study at weekly frequency done for the 1945–1946 Hungarian episode (Engsted, 1998).

Accordingly, we shall test first if money (m) and exchange rate (e) are $I(2)$ processes, and whether they cointegrate such that real money balances ($m - e$) are $I(1)$ process. If so, one can proceed to estimate the Cagan money demand above by testing for cointegration between real balances ($m - e$) and exchange rate depreciation (Δe), and obtaining the cointegrating vector. Alternatively, a semi-elasticity of money demand (α) estimate can be obtained from the cointegrating vector between real balances ($m - e$) and money growth (Δm). The latter cointegration precludes the presence of bubbles and indicates forward looking behavior (Engsted, 1994).

As explained in Section 2 the sample runs from July 1 to December 31, 1993 and covers five working days per week. Table 4 summarizes the results on unit root testing.

Reported results do indeed confirm that both money (m) and exchange rate (e) are $I(2)$ processes, and hence respective first differences Δm and Δe are $I(1)$ processes. Thus the obtained ADF statistics show that the null hypothesis stating that m and e are $I(2)$ cannot be rejected, while the one stating that they are $I(3)$ can. The KPSS test also gives the same results. Namely, the null that m and e are $I(2)$ against

[8] Mark and Sul conclude that an explanation for their results "… may be that the long-run nominal exchange rate is determined directly by monetary fundamentals and not by relative price levels" (Mark and Sul, 2001, p. 47).

Table 5
Testing for the break in currency depreciation (Δe) and money growth (Δm): the Zivot–Andrews unit root test (Period: July 1–December 31, 1993).

	Break in the mean of trend function		Break in the slope of trend function		Break in the mean and slope of trend function	
	Minimum ADF	Date of minimum value	Minimum ADF	Date of minimum value	Minimum ADF	Date of minimum value
Δe_t	−2.79	December, 6, 1993	−3.12	October 20, 1993	−3.11	November 1, 1993
Δm_t	−3.01	December, 6, 1993	−3.79	November 1, 1993	−3.79	December 6, 1993

The 5% critical values for the Zivot–Andrews ADF tests are −4.80, −4.42 and −5.08, respectively, for the model that allows for the break in the mean, in the slope and in the both. There are nine lags in the model for Δe_t and eight in the model for Δm_t.

the alternative of being $I(3)$ is not rejected, while the null that m and e are $I(1)$ is rejected in favor of the alternative that they are $I(2)$. Additionally, both ADF and KPSS tests show that money (m) and exchange rate (e) cointegrate making real money balances ($m - e$) an $I(1)$ process (cf. Table 4).

Furthermore, these processes are not explosive. Namely, the right tail critical value of the ADF test can be used to test whether the processes ($m - e$), Δm and Δe are $I(1)$ or explosive. Since all calculated values of the ADF test are less than the corresponding 5% right tail critical value, we may conclude that the time series considered do not contain an explosive root. Specifically, the ADF test for Δm and Δe, being −1.03 and −1.23 respectively are lower than matching 5% right tail (−0.90); in case of real money balances ADF test (−3.02) is also lower than this critical value.

As shown in Section 2 (cf. Figs. 1 and 2, and Table 1) currency depreciation (Δe) and money growth (Δm) sharply increased by the end of November, suggesting that these series might have experienced a break in their respective means. In order to examine this, the Zivot and Andrews (1992) unit root test is used while testing whether depreciation (Δe) and money growth (Δm) are respectively unit root processes or stationary variables with a single break. Thus we tested for the break in the mean (intercept) of these series, but also for the break in their slopes and then in both, at the unknown point in time. The sample covers the whole period, i.e. July 1–December 31, 1993, and the results are reported in Table 5.

Because in each case explored in Table 5 the minimum value of the ADF test is greater than the corresponding 5% critical value, one cannot reject the null hypothesis that (Δe) and (Δm) are unit root processes, implying further that they do not exhibit a break.

The results reported in Tables 4 and 5 clear the way for estimating the Cagan money demand. Namely, one can proceed and explore cointegration of real money balances ($m - e$) with exchange rate depreciation (Δe) and money growth (Δm) respectively. The results are presented in Table 6.[9]

In both cases, tests employed do confirm the presence of cointegration. Neither process is explosive, i.e. there is no root larger than one, which confirms the results reported above. Alternative semi-elasticity estimates are very close to each other: 5.00 and 5.37. Thus the estimated Cagan money demand seems to be stable even through December 31, hence encompassing the period of the great surge in money growth and exchange rate depreciation as well as the rise in their volatility.

Nevertheless we formally examined the stability of the Cagan money demand using diagnostic tests based on recursive estimation of a cointegrated VAR model. The model explored is the one with money growth (Δm)[10] and results are depicted in Figs. 4 and 5. Fig. 4 shows the test statistics for the max test for constancy of cointegration parameters advocated by Hansen and Johansen (1999),[11] with simulated critical values reported in Dennis (2006).

Values of the test statistics are divided by the 5% critical value, and hence the obtained magnitudes that are less than one suggest parameter stability. As all of them are below one (see Fig. 4) we cannot reject the null hypothesis that the cointegration parameters are constant for the whole sample. Specifically, although the test statistics spike by the end of November, as could be expected (cf. Figs. 1

9 The VAR model that is used for cointegration testing is reported in the Appendix.
10 The same results are obtained for the version with currency depreciation (Δe).
11 It is the LM type test for parameter constancy originally suggested by Nyblom (1989).

Table 6
Cointegration test and estimation of cointegrating vectors (period: July 1–December 31, 1993).

Cointegration between $(m-e)$ and Δe					
Rank	Eigenvalue	Trace test	Cointegrating vector		
			$m_t - e_t$	Δe_t	1
$r=0$	0.145	22.17	1	5.00	−4.13
$r\leq 1$	0.028	3.37		(0.38)	(0.07)
The largest roots of the companion matrix ($r=1$): 1.00, 0.94, 0.94, 0.94, 0.93, 0.93, 0.91, 0.91, 0.89, 0.89.					
Cointegration between $(m-e)$ and Δm					
Rank	Eigenvalue	Trace test	Cointegrating vector		
			$m_t - e_t$	Δm_t	1
$r=0$	0.168	24.09	1	5.37	−4.13
$r\leq 1$	0.010	1.28		(0.42)	(0.09)
The largest roots of the companion matrix ($r=1$): 1.00, 0.89, 0.89, 0.85, 0.85, 0.78, 0.78, 0.77, 0.76, 0.76					

A constant term is restricted to entering a cointegration vector only. There are eight lags in the VAR model of $(m-e)$ and Δm, and twelve lags in the VAR model of $(m-e)$ and Δe. Dummy variables are included in the VAR to model outliers that are identified as extreme values of standardized VAR residuals. The VAR model of $(m-e)$ and Δm contains the following dummy variables: D1, D2, D3, D4, D5, D6, D7, D8 and D9. These dummy variables are defined as follows: D1 = 1 for 1993:07:26 and 0 otherwise, D2 = −1 for 1993:10:1, 1993:10:4 and 0 otherwise, D3 = 1 for 1993:10:11 and 0 otherwise, D4 = 1 for 1993:11:1 and 0 otherwise, D5 = 1 for 1993:09:13 and 0 otherwise, D6 = 1 for 1993:11:29 and 0 otherwise, D7 = 1 for 1993:12:6 and 0 otherwise, D8 = 1 for 1993:12:16 and 0 otherwise and D9 = 1 for 1993:12:24, −1 for 1993:12:27 and 0 otherwise. The VAR model of $(m-e)$ and Δe contains the following dummy variables: D1, D2, D4, D6, D8 and D10. Dummy variable D10 is defined as follows: D10 = 1 for 1993:11:17, 1993:11:18 and 0 otherwise. The 5% critical values for the trace test are simulated with 10000 replications using CATS in RATS 2.0 (Dennis, 2006). In the VAR model of $(m-e)$ and Δm, the 5% critical values are: 19.93 for $r=0$ and 9.19 for $r\leq 1$. In the VAR model of $(m-e)$ and Δe, the 5% critical values are: 20.33 for $r=0$ and 9.31 for $r\leq 1$. Standard errors of estimated cointegration parameters are given in parentheses.

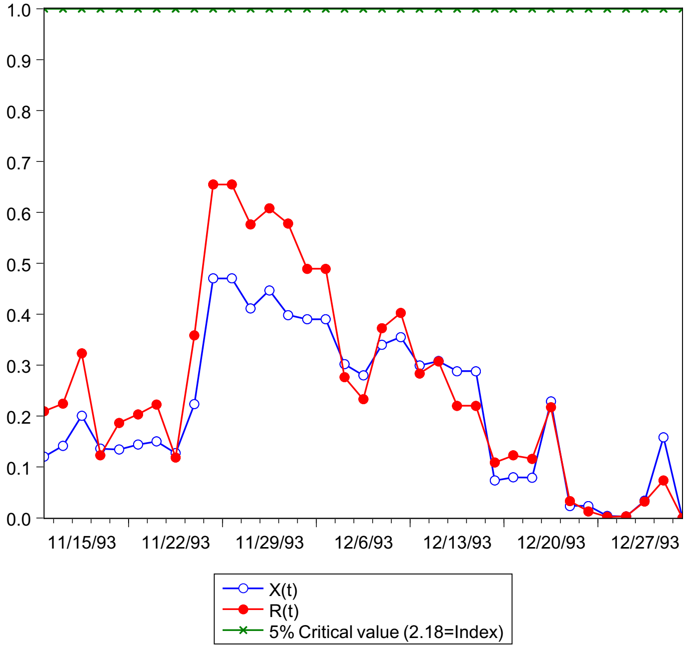

Fig. 4. Recursively computed max test of constant cointegration parameters. X stands for the model with the original variables while R denotes results based on variables corrected for short-term dynamics and interventions (cf. Juselius, 2006).

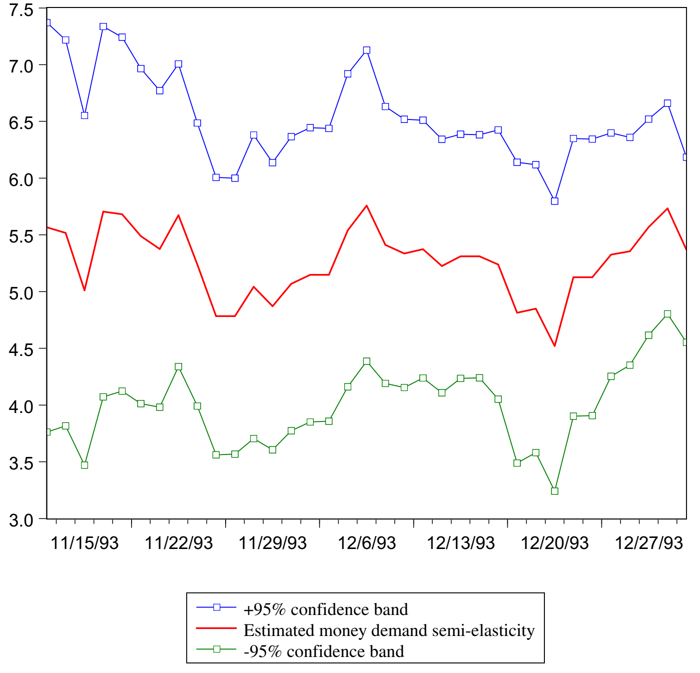

Fig. 5. Recursively computed semi-elasticity estimates. Version of the Cagan model with money growth (Δm) is used. Model with original variables is employed (cf. note to Fig. 4).

and 2, and Table 1), they are still well below the critical value hence indicating the constancy of the Cagan money demand parameters.

In accordance with the results above, the semi-elasticity of money demand also turns out to be quite stable over the turbulent months of November and December. This is demonstrated by its recursively computed estimates depicted in Fig. 5.

In summary, the Cagan money demand is accepted at daily frequency for the most severe period of the Serbian hyperinflation since real money balances ($m - e$) cointegrate with currency depreciation (Δe) and money growth (Δm) respectively, and the estimated relations prove to be stable. The acceptance of this money demand is independent of expectations formation, and may encompass both adaptive and rational expectations.

4. Cagan's paradox revisited

We shall now look at how the Cagan model, specifically the estimated money demand semi-elasticity, concurs with the observed pattern of inflation tax, i.e. its bell shape with non-decreasing inflation tax through November 1993 and subsequent drop in December and January (cf. Table 3).

Following Drazen (1985), the standard measures of the revenues arising from inflation in our case would be, respectively, monetary growth Δm and depreciation rate Δe multiplied by real money holdings ($m - e$).

As to the former (Δm), the corresponding estimate of money demand semi-elasticity is 5.37 (cf. Table 6), implying that the money growth rate of 18.6% per day maximizes revenue from inflation. The actual rate of monetary expansion (cf. Table 1) was below the maximizing one through October, reached it in November (18%), and surpassed the maximizing rate in December and January. The latter suggests that through November the economy was on the increasing side of the Laffer curve and subsequently, in December and January, on the decreasing side. The same result is obtained for the alternative measure that employs exchange rate depreciation (Δe). Specifically, the corresponding semi-elasticity estimate is 5.00 (cf. Table 6), leading to a Laffer's curve maximizing rate of 20% per month. The comparison with the actual average currency depreciation rates (cf. Table 1) in November (21.5%) and December (36.4%) shows again that the economy reached the maximum of the Laffer curve in November, and then switched to the wrong, decreasing side of the curve.

Looking more closely i.e. at the weekly data, one finds that monetary expansion exceeded the maximizing rate (18.6%) in the week of November 15–21 (23% per day), then dropped below it in the subsequent week (10.5%), and finally surpassed it in the week of November 29–December 5 (35.5%). The same pattern, only lagging by one week, is observed for the alternative measure, i.e. inflation tax. Thus the actual rate (27.1%) conclusively exceeded the maximizing rate (20%) in the week of December 6–12.

Accordingly the estimated Cagan model suggests that the maximum of the Laffer curve is attained sometime at the end of November or in early December 1993. So the economy was on the increasing side of the Laffer curve through the end of November and on the wrong side only for the subsequent, last 2 months (seven weeks) of hyperinflation. This pattern concurs with the actual evolution of seigniorage and inflation tax as shown in Section 2. It follows that daily data estimates clearly reject the Cagan's paradox.

The daily results above diverge from those obtained at monthly frequency for the Serbian episode. Thus even the monthly estimates that allow for decreasing semi-elasticity of money demand, although pointing in the right direction nevertheless fall short of resolving Cagan's paradox. Namely, the estimated Barro's (1970, 1972) model for the Serbian episode at monthly frequency suggests that the economy was on the wrong side of the Laffer curve for the last 6 months of hyperinflation,[12] while the corresponding estimate of Easterly et al. (1995) schedule positions the Serbian economy on the decreasing side of Laffer curve for as long as 14 months.[13] These results contradict both the actual inflation tax evolution depicted in Table 3, and daily data estimates. Thus at monthly frequency even decreasing semi-elasticity of money demand cannot capture the true hyperinflation dynamics, specifically throughout its last extreme portion.

Confronting our daily data results with comparable Cagan money demand estimates at monthly frequency for other hyperinflations points to stark differences (see Table 7).

As shown in Table 7 monthly semi-elasticity estimates vary from 3 to 6, even 8, while our daily estimates (5.00–5.37) expressed at monthly frequency are as small as 0.17–0.18. Moreover, the reported monthly studies of hyperinflation do support Cagan's paradox, positioning economies on the decreasing segment of the Laffer curve for the significant period of hyperinflation. Namely, as shown in Table 7, the average inflation rate is higher than the one that maximizes the Laffer curve in all but one case, and statistical tests confirm this (Taylor, 1991). These studies hence imply that in hyperinflation governments continue expanding the money supply at an increasing rate even when the resulting seigniorage is declining, thus creating Cagan's paradox. Namely, the puzzle is why governments opt for higher rates of money growth and inflation when they can collect the same amount of seigniorage at the lower rates.

Contrary to these monthly studies, analysis at daily frequency plainly rejects Cagan's paradox, i.e. the results indicate that the government succeeded in collecting non-decreasing seigniorage almost through the end of hyperinflation. This explains why the Serbian government carried on with money creation at an accelerating rate throughout the episode. Thus daily data findings clearly demonstrate that hyperinflation is triggered and driven by government's need for seigniorage. As opposed to monthly studies, daily results can also explain why hyperinflation lasts as long as it does, by showing that governments continue expanding the money supply at an increasing rate up to the point when seigniorage starts collapsing. In broad picture, the results at daily frequency strongly support the fiscal view of hyperinflation, stating that governments revert to money printing to cover

[12] The results are available from the authors upon request.

[13] Cf. Petrović and Mladenović (2000).

Table 7
Estimated semi-elasticity of money demand in across hyperinflations.

Per month	Semi-elasticity	Average monthly inflation rate		Seigniorage-maximizing inflation rate
		Discrete[a]	Continuous[a]	
Austria	3.8	47%	38.5%	26.3%
Germany	5.3	322	144	18.9
Hungary I	8.3	46	37.8	12.0
Poland	3.4	81	59.3	29.4
Taiwan	4.7	22	19.9	21.3
Greece	3.0	365	154	33.3
Russia	3.1	57	45.1	32.3
Serbia (Δe)	3.4	58.3	45.9	29.4
Germany (Δe)	6.1	28.9	25.4	16.4

Average monthly inflation rates for the six hyperinflations in the 1920s are taken from Cagan (1956), (Table 1) and they respectively cover the hyperinflation periods determined by Cagan, and not the samples used for estimation. Semi-elasticity estimates for Austria, Germany, Hungary I and Poland are from Taylor (1991); for Greece and Russia from Engsted (1994). The remaining two studies are exchange rate models: for Serbia, the sample includes the year 1991 preceding hyperinflation and runs through June 1993, i.e. short of the last 7 months of extreme hyperinflation, Petrović and Mladenović (2000); for Germany the pre-hyperinflation period is also included in the sample, Engsted (1996).

[a] Cf. footnote 6.

Table 8
Misspecification tests in the VAR of $(m-e)$ and Δm (period: July 1–December 31, 1993).

Multivariate tests	
Residual autocorrelation: LM_1, CHISQ(4)	1.89 (p-value = 0.76)
Residual autocorrelation: LM_4, CHISQ(4)	1.59 (p-value = 0.81)
Normality: CHISQ(4)	8.35 (p-value = 0.08)

Univariate tests					
Equation	ARCH(8)	Skewness	Kurtosis	Normality(2)	R^2
$\Delta(m-e)$	5.63	−0.19	4.19	9.08	0.68
$\Delta^2 m$	9.44	0.28	3.07	1.81	0.82

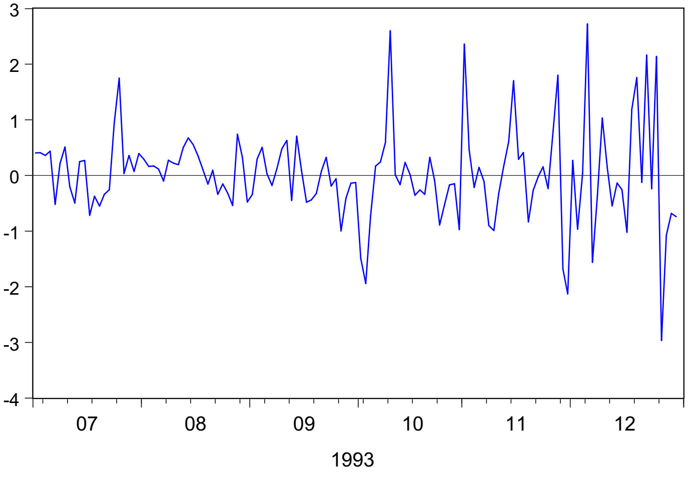

Fig. 6. Estimated cointegration relation between $(m-e)$ and Δm.

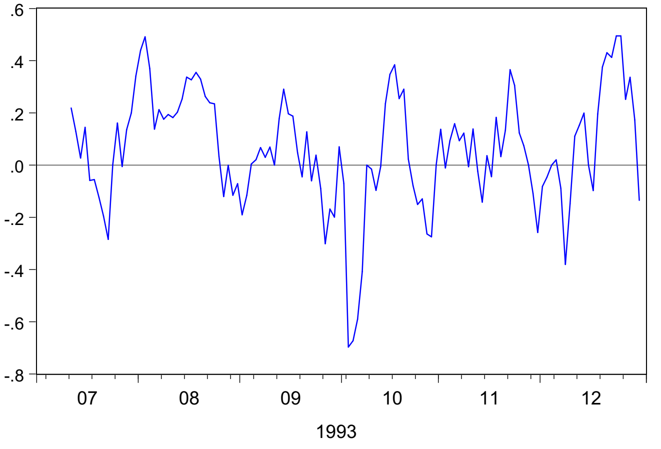

Fig. 7. Estimated cointegration relation between $(m - e)$ and Δm corrected for short-run effects and outliers.

unsustainable fiscal deficits, and that this lasts as long as they succeed in extracting sufficient inflation revenues.

5. Conclusions

The paper offers two main results. First it shows that the Cagan (1956) money demand holds at daily frequency in the extreme portion of Serbia's 1992–1993 hyperinflation, i.e. during its last 6 months. This result strongly supports a conjecture that in advanced hyperinflation the public adjusts their decisions daily, hence calling into question the findings of previous hyperinflation studies done as a rule at monthly frequency. Second, the obtained semi-elasticity estimates reject Cagan's (1956) paradox, by showing that the economy has been on the correct, increasing side of the Laffer curve almost through the end of hyperinflation. The latter result is in sharp contrast with those previously found in monthly studies.

Namely, we obtained semi-elasticity estimates of money demand at daily frequency that are far lower than the those derived from monthly studies. Thus a representative daily semi-elasticity estimate (5.37) expressed at monthly frequency is just 0.18 (0.015 annually), which is about twenty times lower than previous monthly estimates for hyperinflation (cf. Table 7). The inverse of semi-elasticity gives the inflation tax maximizing rate, and it is found to be 559% per month. This is far above the comparable monthly estimates, which imply a maximizing rate in the range of 17–33% per month.

The low value of money demand semi-elasticity is crucial in addressing Cagan's paradox. Thus in the Serbian episode, estimated semi-elasticity implies that money printing and the consequent currency depreciation did not result in the decrease of the inflation tax through the severe period of hyperinflation, except for the last 2 months (seven weeks). The actual inflation tax path concurs with the pattern predicted by the model, both in terms of the bell shape of the Laffer curve and in the timing of the maximum. These results point out that the government's demand for seigniorage forced the Serbian economy into hyperinflation and that it lasted almost as long as non-decreasing seigniorage could be extracted. This resolves Cagan's paradox in the case of the Serbian hyperinflation.

On the other hand, monthly estimates in Serbia (even those allowing for decreasing semi-elasticity) could not capture actual inflation tax evolution, and significantly differ from daily results. These estimates do point in the right direction, i.e. that semi-elasticity decreases as inflation accelerates, however not enough to resolve Cagan's paradox. Thus even varying semi-elasticity money demand schedules (e.g. Barro, 1970; Easterly et al., 1995) at monthly frequency fall short of explaining hyperinflation dynamics and particularly that of inflation tax in the Serbian episode.

The semi-elasticity estimates at daily frequency are robust, i.e. they are obtained from cointegration vectors using a large sample of around 130 daily observations that cover the most extreme portion of hyperinflation. This markedly differs from the previous monthly hyperinflation studies that used only 20–35 observations including relatively low rates of inflation. Both, the small sample and the inclusion of these low rates may have led to inferior monthly estimates and specifically to underestimation of the inflation tax maximizing rate (Barro, 1972).

However, the main reason for the discrepancy between daily and monthly estimates is most probably due to temporal aggregation of essentially daily processes into monthly ones, which when used for modeling may indeed mask true relationships (cf. Taylor, 2001). Namely, we have found that the public adjusts daily in hyperinflation, and consequently monthly sampling might not reveal true processes, implying further that monthly estimates could be unreliable.

The results obtained at daily frequency for the Serbian episode, specifically low semi-elasticity of money demand and the consequent rejection of Cagan's (1956) paradox, may well hold in other hyperinflations. Thus, as in the Serbian case, some monthly studies for other episodes hint that semi-elasticity decreases as inflation accelerates, however not enough to resolve Cagan's paradox (Barro, 1970, 1972; Michael et al., 1994). But even more importantly, the severe portions of a few other episodes, notably those of Germany, Hungary (1945–1946) and Greece, are similar to that of Serbia hence suggesting that the public, in these cases also adjusts at daily frequency. The latter throws doubt on the corresponding monthly estimates and hence on the presence of Cagan's paradox that follows from them.

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Appendix

The VAR models that are used for cointegration testing perform well statistically. This is documented below where some misspecification multivariate and univariate tests of the VAR for $(m - e)$ and Δm are presented. Thus the multivariate tests for residual normality and first and fourth order residual correlation do not suggest misspecification (Table 8). We also reported following univariate tests and statistics based on the estimated residuals from each equation: test for ARCH of order eight, Doornik–Hansen test of normality, estimated coefficients of skewness and kurtosis and R^2. These tests are valid irrespective of whether the variables are $I(0)$, $I(1)$ or explosive, and hence can be performed as a first step independently of subsequent unit root testing (Nielsen, 2006; Engler and Nielsen, 2009). Estimated cointegration relation is depicted in Figs. 6 and 7.

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